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Renormalization of supersymmetric Yang-Mills theories with soft supersymmetry breaking

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Abstract

The renormalization of supersymmetric Yang-Mills theories with soft supersymmetry breaking is performed. As usually done in concrete calculations, the Wess-Zumino gauge is used throughout, and the non-existence of a consistent supersymmetric and gauge invariant regularization is taken seriously. Our central results are a suitable rigorous definition of the models, the correct gauge fixing and ghost terms, the general form of the divergences, and a proof that renormalization of the fields and parameters in the classical Lagrangian yields precisely the correct counterterms to cancel all divergences. In our construction additional spurious parameters appear but are shown to be physically irrelevant. We comment on the inclusion of additional, non-standard, soft breaking terms, and we exemplify our results in the renormalization of supersymmetric QCD.

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1 Introduction

From the beginning, supersymmetric theories have been famous for their extraordinary renormalization properties. Most quadratic divergences are absent, in many theories particular non-renormalization theorems hold, and some are even completely finite [1]. This has stimulated the hope that supersymmetry could play a role in the solution of the hierarchy and naturalness problems [2] and is one of the main motivations for the study of supersymmetric extensions of the standard model, in spite of the absence of any direct experimental evidence [3].

Apart from its fundamental importance, renormalization is necessary to obtain correct results for higher order contributions to physical predictions. With the prospect of future experiments, e.g. at the LHC and a possible linear e^+e^- collider, having control of the quantum corrections is mandatory. These experiments have the potential for decisive tests of supersymmetry at the weak scale, and in order to discriminate between different models the accuracy of the theoretical description has to match the one of the experiments.

Thus it is something of an irony that no completely satisfactory study of the renormalization and renormalizability of supersymmetric extensions of the standard model is available so far. Most of the existing studies are limited to one- or two-loop order and assume the existence of a supersymmetric and gauge-invariant regularization such as dimensional reduction [4]. However, dimensional reduction is mathematically inconsistent [5], and a consistent regularization scheme with the assumed symmetry properties is not known; hence, these studies are not fully conclusive.

One important characteristic of supersymmetric extensions of the standard model is the appearance of so-called soft supersymmetry-breaking terms [6]. Models with soft breaking terms have been renormalized using the Wess-Zumino gauge in ref. [7], but the results cannot be applied directly to phenomenology since for renormalization new kinds of parameters have to be admitted whose physical meaning is unclear.

In this article, the renormalization of supersymmetric non-abelian gauge theories with soft supersymmetry breaking is studied in the formalism commonly used in concrete calculations. The Wess-Zumino gauge is used throughout, and the non-existence of a consistent gauge-invariant and supersymmetric regularization scheme is taken seriously. Our results include

- a definition of the considered models by identities expressing the desired symmetries, in particular gauge invariance and softly-broken supersymmetry,
- a derivation of the correct gauge fixing and ghost terms,
- a proof that all divergences cancel by renormalization of the fields and parameters in the initial classical action without inducing new kinds of interactions,
- a proof that the additional parameters appearing in the course of our construction are spurious and do not influence physical amplitudes.

We restrict ourselves to a simple gauge group and exclude spontaneous symmetry breaking and CP violation.

Together with the treatment of the intricacies of the standard model due to its spontaneously broken, non-semisimple gauge group [8] and supersymmetric non-abelian [9, 10] and abelian [11] gauge theories without soft breaking, this should provide the necessary building blocks for the renormalization of the supersymmetric extensions of the standard model.

The outline of the present article is as follows. In sec. 2 the basic notions of the considered models are introduced, and a discussion of soft supersymmetry breaking is given. In particular, we explain the differences between the breaking terms found by Girardello and Grisaru [6] and the additional non-standard ones mentioned e.g. in [12]. We restrict the soft breaking terms to the GG class. In sec. 3 the symmetry identities describing gauge invariance and softly broken supersymmetry are constructed. The basic idea how to incorporate the GG soft breaking terms is to render them supersymmetric by coupling them to an external chiral supermultiplet as done originally in [6]. Then a Slavnov-Taylor identity of the same structure as in the case with unbroken supersymmetry [9, 10] can be used. In sec. 4 we show that by introducing the external chiral multiplet an infinite number of parameters appears in the most general classical action. That these parameters are all physically irrelevant and do not even appear in practice is demonstrated in sec. 5. The theorems proven there are our central results and finally also imply that all divergences can be absorbed in accordance with the symmetries.

In sec. 6 we consider two alternative approaches and their relation to our work. First, the Slavnov-Taylor identity for softly broken supersymmetry used in [7] is compared to ours and its advantages and disadvantages are discussed. Second, we return to the additional, non-GG soft breaking terms and indicate how the renormalization can be performed when these terms are admitted. Finally, in sec. 7 we apply our results to the renormalization of supersymmetric QCD. In particular, we compare different possibilities to renormalize the squark mixing matrix.

In the appendix our conventions are collected.

2 The model and its symmetries

2.1 Supersymmetric part

We consider supersymmetric Yang-Mills theories with a simple gauge group, coupled to matter. In this class of models we allow for the following fields:

- One Yang-Mills multiplet in the adjoint representation of the gauge group. This multiplet consists of the spin-1 gauge fields A_a^μ and the spin-$\frac{1}{2}$ gauginos $\lambda_a^\alpha, \overline{\lambda}_{a\dot\alpha}$.
- Chiral supersymmetry multiplets (ϕ_i, ψ_i^α) for the matter fields consisting of scalar and spin-$\frac{1}{2}$ fields that transform under a representation of the gauge group which is in general reducible. The corresponding hermitian generators are called T_{ij}^a.

For later use let us introduce the gauge covariant derivative

$$D_\mu = \partial_\mu + igT^a A_a^\mu, \tag{1}$$

where in the adjoint representation T^a has to be replaced by $-if^{abc}$ defined by $[T^a, T^b] = if^{abc}T^c$, and the field strength tensor

$$igT^a F_a^{\mu\nu} = [D^\mu, D^\nu] \,, \tag{2}$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - gf^{abc} A_b^\mu A_c^\nu \,. \tag{3}$$

We use this minimal set of fields corresponding to the Wess-Zumino gauge throughout the whole paper. Still it will be convenient to have at hand the compact superspace notation. In superspace we define the vector superfields in the Wess-Zumino gauge

$$\begin{aligned} V_a(x,\theta,\overline{\theta}) = {} & \theta\sigma^\mu\overline{\theta} A_{a\mu}(x) + i\theta\theta\overline{\theta\lambda}_a(x) - i\overline{\theta\theta}\theta\lambda_a(x) \\ & + \frac{1}{2}\theta\theta\overline{\theta\theta} D_a(x) \,. \end{aligned} \tag{4}$$

Using the covariant derivatives in superspace

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i_\alpha(\sigma^\mu\overline{\theta})\partial_\mu, \tag{5}$$

$$\overline{D}_{\dot\alpha} = \frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} + i(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu, \tag{6}$$

we can define the field strength superfields ($V = T^a V_a$, $W_\alpha = T^a W_{a\alpha}$, $\overline{W}_{\dot\alpha} = T^a \overline{W}_{a\dot\alpha}$)

$$W_\alpha = -\frac{1}{8g}\overline{DD}(e^{-2gV}D_\alpha e^{2gV}) \ , \tag{7}$$

$$\overline{W}_{\dot\alpha} = -\frac{1}{8g}DD((\overline{D}_{\dot\alpha}e^{2gV})e^{-2gV}) \ . \tag{8}$$

The matter fields are combined in chiral superfields

$$\Phi_i(y,\theta) = \phi_i(y) + \sqrt{2}\ \theta\psi_i(y) + \theta\theta F_i(y) \tag{9}$$

with the chiral coordinate $y^\mu = x^\mu - i\theta\sigma^\mu\overline{\theta}$. Whenever we use a superspace expression it is understood that the auxiliary fields D_a and F_i are eliminated by their respective equations of motion derived from the complete classical action, $\frac{\delta\Gamma_{\rm cl}}{\delta D_a} = \frac{\delta\Gamma_{\rm cl}}{\delta F_i} = \frac{\delta\Gamma_{\rm cl}}{\delta F_i^\dagger} = 0$. Using this notation and superspace integrals with the normalization

$$\int d^2\theta\ \theta\theta = \int d^2\overline{\theta}\ \overline{\theta\theta} = 1 \ , \tag{10}$$

the supersymmetric part of the classical action reads

$$\Gamma_{\rm susy} = \int d^4x\, d^2\theta\, d^2\overline{\theta}\ \Phi^\dagger e^{2gV}\Phi + \Big(\int d^4x\, d^2\theta\ \frac{1}{4}W_a^\alpha W_{a\alpha} + W(\Phi) + h.c.\Big) \tag{11}$$

with the superpotential

$$W(\Phi) = \frac{m_{ij}}{2}\Phi_i\Phi_j + \frac{g_{ijk}}{3!}\Phi_i\Phi_j\Phi_k \ . \tag{12}$$

2.2 Soft supersymmetry breaking

Supersymmetry is very appealing from a theoretical point of view but certainly not an exact symmetry of nature. Therefore, for model-building it is important to find breaking terms that do not destroy its attractive features. Such terms—called soft breaking terms—have been found and classified by Girardello and Grisaru (GG) [6]. Their list of soft breaking terms is quite short:

- mass terms for scalar fields: $-M_{ij}^2\phi_i^\dagger\phi_j$,
- holomorphic bilinear and trilinear terms in the scalar fields: $-(B_{ij}\phi_i\phi_j + A_{ijk}\phi_i\phi_j\phi_k + h.c.)$,
- mass terms for gauginos: $\frac{1}{2}\,(M_\lambda\lambda_a\lambda_a + h.c.)$.

These GG terms have two crucial properties: First, they break supersymmetry without introducing quadratic divergences [6]. And second, they may be viewed as a part of a *power-counting renormalizable and supersymmetric* interaction term with an external supermultiplet (spurion). This can be shown by introducing one external chiral multiplet with R-weight 0, mass dimension 0 and a constant shift in its $\hat{f}$ component[2]:

$$\eta(y,\theta) = a(y) + \sqrt{2}\theta\chi(y) + \theta\theta\hat{f}(y), \tag{13}$$

$$\hat{f}(y) = f(y) + f_0. \tag{14}$$

Then the supersymmetric extensions of the above soft breaking terms can easily be written in superspace:

$$\begin{aligned}\Gamma_{\text{soft}} = & -\int d^4x\, d^2\theta\, d^2\overline{\theta}\ \tilde{M}^2_{ij}\eta^\dagger\eta\Phi_i^\dagger\Phi_j \\ & -\int d^4x\, d^2\theta\ (\tilde{B}_{ij}\eta\Phi_i\Phi_j + \tilde{A}_{ijk}\eta\Phi_i\Phi_j\Phi_k) + h.c. \\ & -\int d^4x\, d^2\theta\ \frac{1}{2}\tilde{M}_\lambda\eta W^\alpha_a W_{a\alpha} + h.c.\end{aligned} \tag{15}$$

As long as η and its component fields are treated as external fields with arbitrary values, these interaction terms are manifestly supersymmetric. Only in the limit

$$\begin{aligned}a(x) &= 0, \\ \chi(x) &= 0, \\ f(x) &= 0, \\ \eta(x,\theta) &= \theta\theta f_0,\end{aligned} \tag{16}$$

they reduce to the soft breaking terms with $\tilde{M}^2_{ij}|f_0|^2 = M^2_{ij}$, $\tilde{B}_{ij}f_0 = B_{ij}$, $\tilde{A}_{ijk}f_0 = A_{ijk}$, $\tilde{M}_\lambda f_0 = M_\lambda$.

The GG soft breaking terms comprise all possible terms of mass dimension 2 but not all possible terms of mass dimension 3. Obviously, not only $\lambda\lambda$ and $\phi\phi\phi$ but also $\psi\psi$ and $\phi^\dagger\phi\phi$ are supersymmetry-breaking terms of mass dimension 3. For instance, in the case of the minimal supersymmetric standard model the $\phi\phi\phi$ GG terms are (we adopt the conventions of ref. [12])

$$m_{10}\lambda_t H_2 Q\bar{t} + m_8\lambda_b H_1 Q\bar{b} + m_6\lambda_\tau H_1 L\bar{\tau}\ , \tag{17}$$

whereas the following non-GG terms are also perfectly gauge-invariant supersymmetry-breaking terms:

$$m_9\lambda_t H_1^* Q\bar{t} + m_7\lambda_b H_2^* Q\bar{b} + m_5\lambda_\tau H_2^* L\bar{\tau}\ . \tag{18}$$

The terms of the form $\psi\psi$ and $\phi^\dagger\phi\phi$ are excluded from the GG class because they cannot be extended to a power-counting renormalizable and supersymmetric interaction such as in (15) and in general they introduce quadratic divergences. However, as explained in [12], in many concrete models, like the minimal supersymmetric extension of the standard model, these quadratic divergences are absent. Therefore, concerning only the quadratic divergences, the GG class is too narrow.

[2]The $\hat{f}$ component of this external chiral superfield need not be eliminated.

Turned around, this means that the possible supersymmetric coupling to the spurion η is the more profound characterization of the GG soft breaking terms. And it is also the more far-reaching one: In a manifestly supersymmetric formulation in superspace the GG terms do not change the basic structure of the UV divergences, in particular all non-renormalization theorems still hold in the case with soft breaking, and all divergences in the broken case can be derived from the ones in the unbroken case [13]. Moreover, this spurion mechanism appears naturally in the low-energy limit of supergravity models with spontaneous supersymmetry breaking (see e.g. [14]). With this in mind, for the largest part of the paper we restrict ourselves to the GG soft breaking terms.

In later course we will define the model by its symmetries. Therefore it is important that the coupling to η provides an unambiguous characterization of the GG breaking terms. Our basic ansatz is to introduce them in a supersymmetric way as in eq. (15). We define and renormalize the model in the presence of the external fields a, χ and $\hat{f} = f + f_0$, and we consider physical amplitudes to be calculated in the limit (16). We will return to the additional terms of [12] in section 6.2.

2.3 Quantum numbers

Apart from softly broken supersymmetry and gauge invariance we require that the considered model is invariant under CP conjugation and continuous R-transformations with suitably chosen R-weights for the matter fields. There may be also further global symmetries such as lepton or baryon number conservation, but these we leave unspecified. The relevant quantum numbers of the fields are listed in tab. 1.[3]

For simplification we impose certain restrictions on the matter representation that are satisfied in many models of phenomenological interest—in particular in the supersymmetric extensions of the standard model.

- The matter representation does not contain singlets or the adjoint representation, so the gauginos cannot mix with some of the ψ_i.
- Either by global gauge invariance or by some additional global symmetry such as R-symmetry, mixing between scalar fields and hermitian conjugate scalar fields is forbidden:

$$\begin{aligned}&Z_{ij}^{(1)}\phi_i\phi_j,\ Z_{ij}^{(2)}\hat{f}\phi_i\hat{f}\phi_j \text{ not invariant unless}\\&Z_{ij}^{(1)} = Z_{ij}^{(2)} = 0.\end{aligned}$$

The generalization to other cases is obvious but leads to the appearance of additional mixing matrices that have to be renormalized.

3 Quantization

3.1 BRS transformations, gauge fixing and ghost terms

Perturbative quantization and renormalization in quantum field theories is governed by the fundamental physical requirements of causality and unitarity of the S-matrix [15]. These lead

[3] Due to the way we introduce the soft breaking, generally it is no problem to choose the R-weights in such a way that R-invariance holds as long as $a, \chi, f \neq 0$, even in the supersymmetric standard model. In the limit (16), however, R-invariance is broken, and either only R-parity invariance survives or even R-parity is broken.

χ	A_a^μ	λ_a^α	ϕ_i	ψ_i^α	a	χ^α	$\hat{f}$	c_a	ϵ^α	ω^ν	$\bar{c}_a$	B_a
R	0	1	n_i	n_i-1	0	-1	-2	0	1	0	0	0
Q_c	0	0	0	0	0	0	0	$+1$	$+1$	$+1$	-1	0
GP	0	1	0	1	0	1	0	1	0	1	1	0
dim	1	$3/2$	1	$3/2$	0	$1/2$	1	0	$-1/2$	-1	2	2

Table 1: Quantum numbers. R, Q_c, GP, dim denote R-weight and ghost charge, Grassmann parity and the mass dimension, respectively. The R-weights n_i of the chiral multiplets are left arbitrary. The quantum numbers of the external fields Y_i introduced in sec. 3 can be obtained from the requirement that $\Gamma_{\rm ext}$ is neutral, bosonic and has $dim = 4$. The commutation rule for two general fields is $\chi_1\chi_2 = (-1)^{GP_1 GP_2}\chi_2\chi_1$.

to the existence and the Feynman rules of higher order corrections but determine only the non-local and imaginary parts, respectively. Hence, at each order one can add local and hermitian counterterms, which is the basic ambiguity of the perturbation series and the source for the divergences.

In order to quantize our model perturbatively, a set of symmetry identities has to be found that expresses the desired properties of the model and constitutes a unique definition—or equivalently a unique prescription for the counterterms. By requiring that these symmetry identities are satisfied at each order by the renormalized effective action

$$\Gamma \;=\; \Gamma_{\rm cl} + \mathcal{O}(\hbar)\ , \tag{19}$$

the quantum extension of the classical action, we obtain an algebraic definition that is valid for any regularization scheme. In particular, for such a definition we need a Slavnov-Taylor identity expressing gauge invariance and softly broken supersymmetry. Furthermore, gauge fixing and corresponding ghost terms have to be introduced without interfering with the Slavnov-Taylor identity in order not to spoil unitarity of the physical S-matrix. This is most conveniently done in the BRS formalism [16] (for generalizations to $N = 1$ supersymmetric models in the Wess-Zumino gauge see [9, 10, 7, 11]).

We combine the gauge and supersymmetry transformations and translations into one single BRS operator s by introducing ghost fields $c_a(x)$, $\epsilon^\alpha, \overline{\epsilon}^{\dot\alpha}$ and ω^ν corresponding to gauge and supersymmetry transformations and translations, respectively[4]. Only the Faddeev-Popov ghosts c_a are quantum fields, whereas the other ghosts are space-time independent constants because the corresponding symmetries are global. On the physical fields (i.e. fields carrying no ghost number) the BRS transformations are the sum of gauge and supersymmetry transformations and translations, where the transformation parameters have been promoted to the ghost fields:

$$\begin{aligned} sA_\mu &= \partial_\mu c - ig[c, A_\mu] + i\epsilon\sigma_\mu\overline{\lambda} - i\lambda\sigma_\mu\overline{\epsilon} \\ &\quad - i\omega^\nu\partial_\nu A_\mu\ , \end{aligned} \tag{20}$$

$$\begin{aligned} s\lambda^\alpha &= -ig\{c, \lambda^\alpha\} + \frac{i}{2}(\epsilon\sigma^{\rho\sigma})^\alpha F_{\rho\sigma} + i\epsilon^\alpha D \\ &\quad - i\omega^\nu\partial_\nu\lambda^\alpha\ , \end{aligned} \tag{21}$$

$$s\overline{\lambda}_{\dot\alpha} \;=\; -ig\{c, \overline{\lambda}_{\dot\alpha}\} - \frac{i}{2}(\overline{\epsilon}\overline{\sigma}^{\rho\sigma})_{\dot\alpha}F_{\rho\sigma} + i\overline{\epsilon}_{\dot\alpha}D$$

[4]The notation is the same as in [11] except for the appearance of auxiliary fields in the BRS transformations.

$$
\begin{aligned}
& - i\omega^\nu \partial_\nu \overline{\lambda}_{\dot\alpha} \ , & (22)\\
s\phi_i &= -igc\,\phi_i + \sqrt{2}\,\epsilon\psi_i - i\omega^\nu\partial_\nu\phi_i \ , & (23)\\
s\phi_i^\dagger &= +ig(\phi^\dagger c)_i + \sqrt{2}\,\overline{\psi}_i\overline{\epsilon} - i\omega^\nu\partial_\nu\phi_i^\dagger \ , & (24)\\
s\psi_i^\alpha &= -igc\,\psi_i^\alpha + \sqrt{2}\,\epsilon^\alpha\, F_i - \sqrt{2}\,i(\overline{\epsilon}\overline{\sigma}^\mu)^\alpha D_\mu\phi_i & \\
& \quad - i\omega^\nu\partial_\nu\psi_i^\alpha \ , & (25)\\
s\overline{\psi}_{i\dot\alpha} &= -ig(\overline{\psi}_{\dot\alpha}c)_i - \sqrt{2}\,\overline{\epsilon}_{\dot\alpha}\, F_i^\dagger + \sqrt{2}\,i(\epsilon\sigma^\mu)_{\dot\alpha}(D_\mu\phi_i)^\dagger & \\
& \quad - i\omega^\nu\partial_\nu\overline{\psi}_{i\dot\alpha} \ , & (26)\\
sa &= \sqrt{2}\,\epsilon\chi - i\omega^\nu\partial_\nu a \ , & (27)\\
sa^\dagger &= \sqrt{2}\,\overline{\chi\epsilon} - i\omega^\nu\partial_\nu a^\dagger \ , & (28)\\
s\chi^\alpha &= \sqrt{2}\,\epsilon^\alpha\hat{f} - \sqrt{2}\,i(\overline{\epsilon}\overline{\sigma}^\mu)^\alpha\partial_\mu a - i\omega^\nu\partial_\nu\chi^\alpha \ , & (29)\\
s\overline{\chi}_{\dot\alpha} &= -\sqrt{2}\,\overline{\epsilon}_{\dot\alpha}\hat{f}^\dagger + \sqrt{2}\,i(\epsilon\sigma^\mu)_{\dot\alpha}\partial_\mu a^\dagger - i\omega^\nu\partial_\nu\overline{\chi}_{\dot\alpha} \ , & (30)\\
sf &= \sqrt{2}\,i\overline{\epsilon}\overline{\sigma}^\mu\partial_\mu\chi \ - i\omega^\nu\partial_\nu f \ , & (31)\\
sf^\dagger &= -\sqrt{2}\,i\partial_\mu\overline{\chi}\overline{\sigma}^\mu\epsilon - i\omega^\nu\partial_\nu f^\dagger \ . & (32)
\end{aligned}
$$

Here we have used $A_\mu = T^a A_{a\mu}$ and similar for λ, $\overline{\lambda}$, $F_{\rho\sigma}$, D, c, $\bar{c}$, B. Again, the auxiliary fields D and $F_i, F_i^\dagger$ are understood to be eliminated by their equations of motion.

The various (anti)commutation relations of the transformations are encoded in the nilpotency equation

$$
s^2 \ = \ 0 + \text{field equations} \tag{33}
$$

if the BRS transformations of the ghosts are given by the structure constants of the algebra and the ghosts have the opposite statistics as required by the spin-statistics theorem [16]:

$$
\begin{aligned}
sc &= -igc^2 + 2i\epsilon\sigma^\nu\overline{\epsilon}A_\nu - i\omega^\nu\partial_\nu c \ , & (34)\\
s\epsilon^\alpha &= 0 \ , & (35)\\
s\overline{\epsilon}^{\dot\alpha} &= 0 \ , & (36)\\
s\omega^\nu &= 2\epsilon\sigma^\nu\overline{\epsilon} \ . & (37)
\end{aligned}
$$

The sum of the gauge fixing and ghost terms has to be BRS invariant in order to ensure the decoupling of the unphysical degrees of freedom and the unitarity of the physical S-matrix. Thus it can be obtained as the BRS transformation of some fermionic expression with ghost number -1. In order to define such an expression we introduce the antighosts $\bar{c}_a(x)$ and auxiliary fields B_a with BRS transformations

$$
\begin{aligned}
s\bar{c} &= B - i\omega^\nu\partial_\nu\bar{c} \ , & (38)\\
sB &= 2i\epsilon\sigma^\nu\overline{\epsilon}\partial_\nu\bar{c} - i\omega^\nu\partial_\nu B \ . & (39)
\end{aligned}
$$

Then we can write down the usual renormalizable gauge fixing term with arbitrary gauge parameter ξ and a linear gauge fixing function $f_a = \partial_\mu A_a^\mu$:

$$
\begin{aligned}
\Gamma_{\text{fix}} &= \int d^4x \ s[\bar{c}_a(f_a + \frac{\xi}{2}B_a)] & \\
&= \int d^4x \Big(B_a f_a + \frac{\xi}{2}B_a^2\Big) + \Gamma_{\text{gh}} \ . & (40)
\end{aligned}
$$

For diagrammatic calculations it is customary to eliminate the auxiliary fields B_a, yielding the usual gauge fixing term $-\frac{1}{2\xi} f_a f_a$. The correct form of the ghost terms contained in $\Gamma_{\rm fix}$ is explicitely:

$$
\begin{aligned}
\Gamma_{\rm gh} \quad &= \quad \int d^4x \Big(-\bar{c}_a \partial_\mu (D^\mu c)_a \\
&\quad - \bar{c}_a \partial^\mu (i\epsilon\sigma_\mu \overline{\lambda}_a - i\lambda_a \sigma_\mu \overline{\epsilon}) + \xi i\epsilon\sigma^\nu \overline{\epsilon} (\partial_\nu \bar{c}_a) \bar{c}_a \Big).
\end{aligned}
\tag{41}
$$

This result coincides with the one for models without soft breaking as treated in [10] in the Landau gauge ($\xi = 0$).

Apparently not only gauge invariance but even supersymmetry is broken by the gauge fixing term. But since the gauge fixing and ghost terms are total BRS transformations, gauge invariance as well as supersymmetry can be maintained on gauge-invariant observables [10]. However, the supersymmetry breaking of the gauge fixing necessitates terms involving the ϵ ghosts, and it is part of the reason for the appearance of loop corrections to the supersymmetry transformations [11].

To summarize, up to now we have constructed the contributions

$$
\Gamma_{\rm susy} + \Gamma_{\rm soft} + \Gamma_{\rm fix} \tag{42}
$$

to the classical action. The first two terms are gauge-invariant and supersymmetric (as long as η is not set to its physical limit (16)), whereas the last term breaks both symmetries. All three terms, however, are invariant under the BRS transformations (20-39).

3.2 Defining symmetry identities

The BRS transformations cannot be used directly in the definition of the quantum theory. Most of the BRS transformations are non-linear in the propagating fields and thus affected by quantum corrections. In order to cope with the renormalization of the composite operators $s\varphi_i$ we couple them to external fields Y_i:

$$
\begin{aligned}
\Gamma_{\rm ext} \quad &= \quad \int d^4x \Big(Y_{A_a^\mu} s A_a^\mu + Y_{\lambda_a}^\alpha s\lambda_{a\alpha} + Y_{\overline{\lambda}_a \dot\alpha} s\overline{\lambda}_a^{\dot\alpha} \\
&\quad + Y_{\phi_i} s\phi_i + Y_{\phi_i^\dagger} s\phi_i^\dagger + Y_{\psi_i}^\alpha s\psi_{i\alpha} + Y_{\overline{\psi}_i \dot\alpha} s\overline{\psi}_i^{\dot\alpha} \\
&\quad + Y_{c_a} s c_a \Big) \ .
\end{aligned}
\tag{43}
$$

Using the external Y fields we can write down the Slavnov-Taylor operator $S(\cdot)$ corresponding to the BRS operator s. Acting on a general functional $\mathcal{F}$ it reads:

$$
S(\mathcal{F}) \quad = \quad S_0(\mathcal{F}) + S_{\rm soft}(\mathcal{F}) \ , \tag{44}
$$

$$
\begin{aligned}
S_0(\mathcal{F}) \quad &= \quad \int d^4x \Big(\frac{\delta\mathcal{F}}{\delta Y_{A_a^\mu}} \frac{\delta\mathcal{F}}{\delta A_a^\mu} + \frac{\delta\mathcal{F}}{\delta Y_{\lambda_a \alpha}} \frac{\delta\mathcal{F}}{\delta \lambda_a^\alpha} + \frac{\delta\mathcal{F}}{\delta Y_{\overline{\lambda}_a}^{\dot\alpha}} \frac{\delta\mathcal{F}}{\delta \overline{\lambda}_{a\dot\alpha}} \\
&\quad + \frac{\delta\mathcal{F}}{\delta Y_{\phi_i}} \frac{\delta\mathcal{F}}{\delta \phi_i} + \frac{\delta\mathcal{F}}{\delta Y_{\phi_i^\dagger}} \frac{\delta\mathcal{F}}{\delta \phi_i^\dagger} + \frac{\delta\mathcal{F}}{\delta Y_{\psi_i \alpha}} \frac{\delta\mathcal{F}}{\delta \psi_i^\alpha} \\
&\quad + \frac{\delta\mathcal{F}}{\delta Y_{\overline{\psi}_i}^{\dot\alpha}} \frac{\delta\mathcal{F}}{\delta \overline{\psi}_{i\dot\alpha}}
\end{aligned}
$$

$$
\begin{aligned}
& + \frac{\delta \mathcal{F}}{\delta Y_{c_a}} \frac{\delta \mathcal{F}}{\delta c_a} + s\bar{c_a} \frac{\delta \mathcal{F}}{\delta \bar{c_a}} + sB_a \frac{\delta \mathcal{F}}{\delta B_a} \Big) \\
& + s\epsilon^\alpha \frac{\partial \mathcal{F}}{\partial \epsilon^\alpha} + s\bar{\epsilon}_{\dot\alpha} \frac{\partial \mathcal{F}}{\partial \bar{\epsilon}_{\dot\alpha}} + s\omega^\nu \frac{\partial \mathcal{F}}{\partial \omega^\nu} \ , \qquad (45)
\end{aligned}
$$

$$
\begin{aligned}
S_{\text{soft}}(\mathcal{F}) \;=\; & \int d^4x \Big(sa \frac{\delta \mathcal{F}}{\delta a} + sa^\dagger \frac{\delta \mathcal{F}}{\delta a^\dagger} + s\chi^\alpha \frac{\delta \mathcal{F}}{\delta \chi^\alpha} \\
& + s\overline{\chi}_{\dot\alpha} \frac{\delta \mathcal{F}}{\delta \overline{\chi}_{\dot\alpha}} + sf \frac{\delta \mathcal{F}}{\delta f} + sf^\dagger \frac{\delta \mathcal{F}}{\delta f^\dagger} \Big) \ . \qquad (46)
\end{aligned}
$$

Now we are in the position to spell out the complete definition of the symmetries of the model as a set of requirements on the effective action Γ, the quantum extension of the classical action Γ_{cl} and the generating functional of one-particle irreducible vertex functions:

- Slavnov-Taylor identity:

$$
S(\Gamma) \;=\; 0 \ . \qquad (47)
$$

  This identity combines the higher order equivalent of the nilpotency relation (33), meaning that the renormalized symmetry transformations still satisfy the desired (anti)commutation relations, and the invariance of Γ under the renormalized symmetry transformations. The violating terms in eq. (33) are absorbed by the appearance of terms in Γ that are bilinear in the Y fields [17].

- Gauge fixing condition:

$$
\frac{\delta \Gamma}{\delta B_a} \;=\; \frac{\delta \Gamma_{\text{fix}}}{\delta B_a} = f_a + \xi B_a \ . \qquad (48)
$$

- Translational ghost equation:

$$
\frac{\delta \Gamma}{\delta \omega^\nu} \;=\; \frac{\delta \Gamma_{\text{ext}}}{\delta \omega^\nu} \qquad (49)
$$

  with Γ_{ext} in eq. (43).

- Global symmetries: We require Γ to be invariant under CP conjugation and under global gauge transformations and continuous R-transformations and to preserve ghost number. There may be further symmetries such as lepton number conservation, but these we leave unspecified.

- Physical part: As already stated in sec. 2.2, the physical part of the effective action is defined to be

$$
\Gamma|_{a=\chi=f=0} \ . \qquad (50)
$$

  In this limit, already defined in eq. (16), supersymmetry is softly broken by GG terms.

For later use we introduce the abbreviation $\text{Sym}(\Gamma) = 0$ for this set of symmetry requirements:

$$
\text{Sym}(\Gamma) = 0 \quad \Leftrightarrow \quad (47), (48), (49), \text{Global symmetries.} \qquad (51)
$$

The whole construction was oriented on the classical action

$$
\Gamma_{\text{cl}} \;=\; \Gamma_{\text{susy}} + \Gamma_{\text{soft}} + \Gamma_{\text{fix}} + \Gamma_{\text{ext}} \ , \qquad (52)
$$

and hence Γ_{cl} is a special solution to these conditions. Note that the implicit elimination of the D_a and $F_i, F_i^\dagger$ fields yields the bilinear terms in the external Y fields alluded to above. A more general solution will be given explicitely in eq. (63).

4 Renormalization I: Basics

The symmetry identities constitute a rigorous definition of the considered models. However, it remains to be checked whether the models defined in this way are renormalizable. This is done in this and the following section.

4.1 Generalized classical solution

In this subsection we assume that the symmetry identities can be established at each order by adding appropriate counterterms. Once the symmetries hold at the order $\hbar^n$, there still may arise divergences and counterterms may be added. Both the divergences and the counterterms cannot interfere with the symmetries, which means that both are of the form Γ_{sym} with

$$\begin{aligned} & \text{Sym}(\Gamma_{\leq \text{n-Loop, regularized}} + \hbar^n \Gamma_{\text{sym}}) \\ = \; & \text{Sym}(\Gamma_{\leq \text{n-Loop, regularized}}) + \mathcal{O}(\hbar^{n+1}) \; , \end{aligned} \tag{53}$$

which reduces to

$$\text{Sym}(\Gamma_{\text{cl}} + \zeta \Gamma_{\text{sym}}) \;=\; \mathcal{O}(\zeta^2) \; , \tag{54}$$

with some arbitrary infinitesimal parameter ζ, since all symmetry identities are linear or bilinear.

A model is renormalizable if all divergences can be absorbed by counterterms corresponding to renormalization of the fields and parameters in the classical action and if the number of physical parameters is finite.

Eq. (54) shows how to find the general structure of the possible divergences and counterterms. Since the perturbed action $\Gamma_{\text{cl}} + \zeta \Gamma_{\text{sym}}$ is a solution of the symmetry identities in terms of a local power-counting renormalizable functional (classical solution), simply the most general of these classical solutions has to be calculated.

In this subsection we determine a certain set of classical solutions with a surprising result. These solutions depend on infinitely many parameters!

One way to obtain solutions different from (52) is obvious. Since η is neutral with respect to all quantum numbers and has dimension 0 we can write

$$\begin{aligned} \Gamma_{\text{susy}} \;=\; & \int d^4x \, d^2\theta \, d^2\overline{\theta} \; r_1(\eta, \eta^\dagger) \Phi^\dagger e^{2gV} \Phi \\ & + \Big(\int d^4x \, d^2\theta \; r_2(\eta) W_a^\alpha W_{a\alpha} + W(\Phi, \eta) + h.c. \Big) \; , \\ \Gamma_{\text{soft}} \;=\; & - \int d^4x \, d^2\theta \, d^2\overline{\theta} \; r_{3ij}(\eta, \eta^\dagger) \Phi_i^\dagger \Phi_j \\ & - \int d^4x \, d^2\theta \; (r_{4ij}(\eta) \Phi_i \Phi_j \\ & \qquad + r_{5ijk}(\eta) \Phi_i \Phi_j \Phi_k) + h.c. \\ & - \int d^4x \, d^2\theta \; r_6(\eta) W_a^\alpha W_{\alpha a} + h.c. \end{aligned} \tag{55}$$

as possible generalizations of (11), (15) that maintains the symmetry properties of Γ_{cl}. Here r_1, r_3 are real functions of $\eta, \eta^\dagger$, and r_2, r_4, r_5, r_6 are holomorphic functions of η (in fact, r_6 is redundant). Expanding $r_1 \ldots r_6$ in a Taylor series leads to infinitely many interaction terms in

$\Gamma_{\rm cl}$. The fact that this generalized action is still symmetric means that to all of these terms there can be divergent loop contributions and that to each of them a normalization condition is needed.

There is a further, more complicated way to perturb the classical action. We can modify the superfields appearing in $\Gamma_{\rm susy}$ and $\Gamma_{\rm soft}$ by terms depending on a, χ, f and modify accordingly the external field part corresponding to the supersymmetry transformations. One specific possibility is the following modification of the chiral superfields parametrized by three arbitrary functions u_1, u_2, u_3 of a and $a^\dagger$:

$$
\begin{aligned}
\Phi_i &= u_{1ij}(a,a^\dagger)\phi_j + \sqrt{2}(u_1u_2)_{ij}(a,a^\dagger)\theta\psi_j \\
&\quad - \sqrt{2}(u_1u_3)_{ij}(a,a^\dagger)\theta\chi\phi_j + \theta\theta F_i \ , \qquad (56) \\
\Gamma_{\rm ext}^{\phi,\psi-\rm Part} &= \int d^4x \Big(Y_{\phi_i}\Big[\sqrt{2}u_{2ij}\epsilon\psi_j - (u_1^{-1}s_\epsilon u_1)_{ij}\phi_j \\
&\quad - \sqrt{2}u_{3ij}\epsilon\chi\phi_j\Big] \\
&\quad - Y_{\psi_i\alpha}\Big[-(u_2^{-1}u_1^{-1}s_\epsilon u_1u_2)_{ij}\psi_j^\alpha \\
&\quad + \sqrt{2}(u_2^{-1}u_3u_2)_{ij}\epsilon\psi_j\chi^\alpha - \sqrt{2}(u_2^{-1}u_3u_3)_{ij}\epsilon\chi\phi_j\chi^\alpha \\
&\quad + (u_2^{-1}u_1^{-1}(s_\epsilon u_1u_3) - u_2^{-1}u_3u_1^{-1}(s_\epsilon u_1))_{ij}\phi_j\chi^\alpha \\
&\quad - \sqrt{2}i(\overline{\epsilon\sigma}^\mu)^\alpha u_{2ij}^{-1}\big(D_\mu\phi_j \\
&\qquad + (u_1^{-1}\partial_\mu u_1)_{jk}\phi_k + u_{3jk}\phi_k\partial_\mu a\big) \\
&\quad + \sqrt{2}\epsilon^\alpha(u_1u_2)_{ij}^{-1}F_j + \sqrt{2}\epsilon^\alpha(u_2^{-1}u_3)_{ij}\phi_j\hat{f}\Big] \\
&\quad + h.c. + \ \text{Terms involving } c,\omega^\nu\Big) \ . \qquad (57)
\end{aligned}
$$

Here s_ϵ denotes only the $\epsilon,\overline{\epsilon}$-dependent part of the BRS transformation. The terms involving c, ω^ν are identical to those in (43). The rather odd form of the modified external field part can be obtained easily from the requirement that Φ_i in (56) transforms again as a chiral superfield. This necessitates modified BRS transformations $s^{\rm mod}\phi$, $s^{\rm mod}\psi$ that are coupled to the Y fields in (57). In contrast, (ϕ_i, ψ_i, F_i) alone do not form a chiral multiplet any more.

Similarly, the vector superfield and the corresponding part of $\Gamma_{\rm ext}$ can be modified as follows:

$$
\begin{aligned}
V &= v_1(a,a^\dagger)(\theta\sigma^\mu\overline{\theta}A_\mu \\
&\quad + i\theta\theta\overline{\theta}(\overline{\lambda}v_2(a,a^\dagger) + \overline{\sigma}^\mu\chi A_\mu v_3(a,a^\dagger)) \\
&\quad - i\overline{\theta\theta}\theta(\lambda v_2(a,a^\dagger) - \sigma^\mu\overline{\chi}A_\mu v_3(a,a^\dagger))) \\
&\quad + \frac{1}{2}\theta\theta\overline{\theta\theta}D \ , \qquad (58) \\
\Gamma_{\rm ext}^{A_\mu,\lambda-\rm Part} &= \int d^4x \Big(Y_{A_{a\mu}}\Big[i\epsilon\sigma_\mu(\overline{\lambda}v_2 + \overline{\sigma}^\nu\chi A_{a\nu}v_3) \\
&\quad - i(\lambda_a v_2 + \overline{\chi\sigma}^\nu A_{a\nu}v_3)\sigma_\mu\overline{\epsilon} - A_{a\mu}(v_1^{-1}s_\epsilon v_1)\Big] \\
&\quad + \Big(-Y_{\lambda_a\alpha}\Big[\frac{i}{2}\epsilon\sigma^{\rho\sigma}(v_1v_2)^{-1}F_{a\rho\sigma}(v_1A) \\
&\quad + i(v_1v_2)^{-1}\epsilon D_a + \sqrt{2}v_3v_2^{-1}\hat{f}^\dagger\overline{\epsilon\sigma}^\mu A_{a\mu} \\
&\quad - (v_1^{-1}v_2^{-1}s_\epsilon v_1v_2)\lambda_a - \big[i\epsilon\sigma_\mu(\overline{\lambda}_a + v_3v_2^{-1}\overline{\sigma}^\nu\chi A_{a\nu}) \\
&\quad - i(\lambda + v_3v_2^{-1}\overline{\chi\sigma}^\nu A_{a\nu})\sigma_\mu\overline{\epsilon}\big]\overline{\chi\sigma}^\mu v_3
\end{aligned}
$$

$$
\begin{aligned}
&- v_3 v_2^{-1}\sqrt{2} i\epsilon\sigma^\nu(\partial_\nu a^\dagger)\overline{\sigma}^\mu A_{a\mu} \\
&- (s_\epsilon v_3) v_2^{-1}\overline{\chi}\overline{\sigma}^\mu A_{a\mu}\Big] + h.c.\Big) \\
&+ \text{ Terms involving } c, \omega^\nu\Big)
\end{aligned} \tag{59}
$$

Here a modified field strength tensor $F_{a\rho\sigma}(v_1 A) = \partial_\rho(v_1 A_{a\sigma}) - \partial_\sigma(v_1 A_{a\rho}) - g f^{abc} v_1^2 A_{b\rho} A_{c\sigma}$ has been introduced.

Note that the functions u_1, u_2, v_1, v_2 are $a, a^\dagger$-dependent generalizations of field renormalizations of the matter and gauge fields. On the other hand, u_3, v_3 are new kinds of parameters corresponding to field renormalizations of the form

$$
\psi \;\to\; \psi - u_3 \chi\phi \;, \tag{60}
$$

$$
\lambda_\alpha \;\to\; \lambda_\alpha - v_3(\sigma^\mu\overline{\chi})_\alpha A_\mu \;. \tag{61}
$$

In addition to these modifications, obviously a field renormalization of the Faddeev-Popov ghost

$$
c \to \sqrt{Z_c}\, c \;, \qquad Y_c \to \sqrt{Z_c}^{-1} Y_c \tag{62}
$$

and renormalization of all parameters appearing in Γ_{cl} in eq. (52) is possible without violating the symmetry identities.

We conclude that the supersymmetry algebra is unstable and allows for arbitrary functions $u_{1,2,3}$ and $v_{1,2,3}$ with again an infinite number of Taylor coefficients that have to be renormalized. So, even without calculating the classical solution to the symmetry identities in full generality, we know that infinitely many normalization conditions are needed and the effective action Γ depends on infinitely many parameters.

Now we can write down a more general classical solution to the symmetry identities than (52) that still has canonically normalized fields but contains the u_3 and v_3 parameters:

$$
\begin{aligned}
&\Gamma_{\text{cl, canonical}}\big|_{a=\chi=0} \\
&\qquad = \Gamma^0_{\text{susy}} + \Gamma^0_{\text{soft}} + \Gamma^0_{\text{ext}} + \Gamma^0_{\text{bil}} + \Gamma_{\text{fix}} \;,
\end{aligned} \tag{63}
$$

$$
\begin{aligned}
\Gamma^0_{\text{susy}} &= \int d^4x \Big(-\frac{1}{4}(F^a_{\mu\nu})^2 \\
&\quad + \frac{i}{2}\overline{\lambda}^a\overline{\sigma}^\mu(D_\mu\lambda)^a + \frac{i}{2}\lambda^a\sigma^\mu(D_\mu\overline{\lambda})^a \\
&\quad + (D^\mu\phi)^\dagger(D_\mu\phi) + \overline{\psi}\overline{\sigma}^\mu i D_\mu\psi \\
&\quad - \sqrt{2}g(i\overline{\psi}\overline{\lambda}\phi - i\phi^\dagger\lambda\psi) \\
&\quad - \left(\frac{1}{2}\psi_i\psi_j\frac{\partial^2 W(\phi)}{\partial\phi_i\partial\phi_j} + h.c.\right) \\
&\quad - \frac{1}{2}(\phi^\dagger g T^a\phi)^2 - \left|\frac{\partial W(\phi)}{\partial\phi_i}\right|^2 \Big) \;,
\end{aligned} \tag{64}
$$

$$
\begin{aligned}
\Gamma^0_{\text{soft}} &= \int d^4x \Big(-\tilde{M}^2_{ij}\hat{f}^\dagger\hat{f}\phi_i^\dagger\phi_j \\
&\quad - \left(\tilde{B}_{ij}\hat{f}\phi_i\phi_j + \tilde{A}_{ijk}\hat{f}\phi_i\phi_j\phi_k + h.c.\right) \\
&\quad + \frac{1}{2}\left(\tilde{M}_\lambda\hat{f}\lambda^a\lambda^a + h.c.\right)\Big) \;,
\end{aligned} \tag{65}
$$

$$
\begin{aligned}
\Gamma^0_{\rm ext} &= \Gamma_{\rm ext}\big|^{D_a\to -g\phi^\dagger T_a\phi}_{F_i\to-(\partial W(\phi)/\partial\phi_i)^\dagger} \\
&\quad + \int d^4x\Big(-Y_{\psi_i\alpha}(\sqrt{2}\epsilon^\alpha\hat f u_{3ij}(0)\phi_j) \\
&\quad - Y_{\lambda_a\alpha}\sqrt{2}v_3(0)\hat f^\dagger\overline{\epsilon\sigma}^{\mu\alpha}A_{a\mu}\Big) + h.c.\ , \qquad (66)\\
\Gamma^0_{\rm bil} &= \int d^4x\Big(\frac{1}{2}(Y_{\lambda_a}\epsilon + Y_{\overline{\lambda}_a}\overline{\epsilon})^2 + 2(Y_{\psi_i}\epsilon)(Y_{\overline{\psi}_i}\overline{\epsilon})\Big)\ . \qquad (67)
\end{aligned}
$$

The part containing the external a and χ fields is suppressed here because its concrete form is not relevant for our discussion, and only the $\hat f$ component of the η multiplet is retained.

4.2 Remarks on anomalies

In the preceding subsection we have assumed that the symmetry identities can be maintained at each order of perturbation theory. In principle this need not be true, because there could be anomalies. For unbroken supersymmetric Yang-Mills theories it is well known that the only possible anomaly is the supersymmetric extension of the chiral gauge anomaly [18, 9, 10]. In particular, the relevant cohomology does not depend on the chiral multiplets at all. In spite of the soft breaking, the formulation of our model is the same as the one for unbroken supersymmetric Yang-Mills theories except for the appearance of the additional chiral η multiplet of dimension 0. Therefore, we assume that our model is anomaly free and the symmetry identities can be restored by suitable counterterms at each order.

However, one also has to check for infrared anomalies, i.e. breakings of the symmetry identities that can only be absorbed by counterterms of infrared dimension less than 4. Using the assignments from [7], in principle counterterms of infrared dimension ≥ 2.5 could show up. Since R-invariance can be assumed to be manifestly preserved only R-invariant counterterms have to be considered, and there are no such counterterms of infrared dimension < 4 that involve at least two propagating fields. The other ones cannot be inserted in higher order loop diagrams and thus are harmless, so there are no infrared anomalies.

5 Renormalization II: Physical part of the model

In general, a model depending on an infinite number of parameters has no predictive power. But this is not necessarily the case here, because all physical amplitudes have to be derived from the effective action Γ in the limit (16), $a = \chi = f = 0$. And we have not yet checked which of the parameters can have any influence on Γ in this limit.

In this section we prove two theorems showing that the infinitely many unwanted parameters are irrelevant for physical quantities and do not appear in practical calculations. Thus the number of physical parameters is finite and the considered models are renormalizable.

In fact, it is easy to see that this should hold at the classical level by comparing the perturbed classical solutions presented above with the original classical action. In the limit (16) all these actions reduce to the same form (52), only modified by field and parameter renormalization and by the appearance of the u_3, v_3 parameters.

Our main idea is to consider the symmetry identities and the effective action in the intermediate limit

$$
a = \chi = 0, f \text{ arbitrary}, \qquad (68)
$$

i.e. requiring only $\mathrm{Sym}(\Gamma)|_{a=\chi=0} = 0$ instead of the full symmetry identities. In this limit the unwanted parameters do not appear but still the symmetry identities are restrictive enough.

The essentials of the two theorems are the following:

1. The only relevant quantities Γ depends on are[5]

   - the field renormalization constants Z_A, Z_λ, Z_c, Z_ϕ, Z_ψ,
   - the gauge coupling g,
   - the parameters in the superpotential m_{ij}, g_{ijk},
   - the soft breaking parameters $\tilde{M}^2_{ij}, \tilde{B}_{ij}, \tilde{A}_{ijk}, \tilde{M}_\lambda$.

   Indeed, suppose Γ_1 and Γ_2 are two solutions of the symmetry identities $\mathrm{Sym}(\Gamma_{1,2}) = 0$ and both satisfy the same normalization conditions for these quantities. Then both solutions differ at most in local terms of the form

$$\begin{aligned} \Gamma_1|_{a=\chi=0} - \Gamma_2|_{a=\chi=0} = \\ \int d^4x \Big(& -Y_{\psi_i\alpha}\sqrt{2}\epsilon^\alpha \hat{f}(u_{3ij}(0) + \delta u_{3ij}(0))\phi_j \\ & - Y_{\lambda\alpha}\sqrt{2}(v_3(0) + \delta v_3(0))\hat{f}^\dagger\overline{\epsilon\sigma}^{\mu\alpha}A_\mu\Big) + h.c. \end{aligned} \tag{69}$$

   These terms are all linear in the propagating fields and involve the Y and ϵ fields which do not appear in physical amplitudes. Thus Γ_1 and Γ_2 are equivalent with respect to their physics content. More details and the proof can be found in subsec. 5.2.

2. In practical calculations it is sufficient to solve the symmetry identities in the limit (68),

$$\mathrm{Sym}(\Gamma)|_{a=\chi=0} = 0\ . \tag{70}$$

   Each of these solutions can be extended to a full solution Γ^{exact} that contains the same physics and satisfies

$$\begin{aligned} \mathrm{Sym}(\Gamma^{\mathrm{exact}}) &= 0\ , & (71)\\ \Gamma|_{a=\chi=0} &= \Gamma^{\mathrm{exact}}|_{a=\chi=0}\ . & (72) \end{aligned}$$

   Since in the evaluation of $\mathrm{Sym}(\Gamma)|_{a=\chi=0}$ the unphysical parameters do not appear one has no need to calculate Feynman rules or vertex functions involving these parameters. This theorem is proven in subsec. 5.1 for the classical level and subsec. 5.3 for the quantum level.

For practical calculations the theorems have an important implication. It is a possible and sufficient prescription to impose only $\mathrm{Sym}(\Gamma)|_{a=\chi=0} = 0$ and require normalization conditions only for the physical parameters listed in theorem 1. Each solution of this prescription is equivalent in physics respects to a full solution of the symmetry identities, and any two solutions differ only in the physically irrelevant part.

The proofs of these theorems are now given in the order of their logical interdependence. First we prove a lemma which is a more general form of theorem 2 on the classical level and introduce some useful notation. The proof of this lemma will show in particular in which sense the limit (68) is special. Then this lemma is used to prove theorem 1 and finally theorem 2 on the quantum level.

[5] Of course the field renormalization constants drop out in S-matrix elements. But their normalization affects the Green functions relevant for physical processes.

5.1 Classical solution and invariant counterterms

Let R be the following operator for a renormalization transformation of all parameters and fields appearing in $\Gamma_{\text{cl, canonical}}|_{a=\chi=0}$ defined in eq. (63):

$$
\begin{array}{rcl}
& R: & \\
\{A^\mu, Y_{A^\mu}, & \to & \{\sqrt{Z_A}A^\mu, \sqrt{Z_A}^{-1} Y_{A^\mu}, \\
B, \bar{c}, \xi\} & & \quad \sqrt{Z_A}^{-1} B, \sqrt{Z_A}^{-1}\bar{c}, Z_A\xi\} \\
\{\lambda, Y_\lambda\} & \to & \{\sqrt{Z_\lambda}\lambda, \sqrt{Z_\lambda}^{-1}\} \\
\{c, Y_c\} & \to & \{\sqrt{Z_c}c, \sqrt{Z_c}^{-1} Y_c\} \\
\{\phi_i, Y_{\phi_i}\} & \to & \{\sqrt{Z_{\phi_{ij}}}\phi_j, \sqrt{Z_{\phi_{ij}}}^{-1} Y_{\phi_j}\} \\
\{\psi_i, Y_{\psi_i}\} & \to & \{\sqrt{Z_{\psi_{ij}}}\psi_j, \sqrt{Z_{\psi_{ij}}}^{-1} Y_{\psi_j}\} \\
\{g, m_{ij}, g_{ijk}\} & \to & \{g + \delta g, m_{ij} + \delta m_{ij}, g_{ijk} + \delta g_{ijk}\} \\
\{\tilde{M}^2_{ij}, \tilde{B}_{ij}, & \to & \{\tilde{M}^2_{ij} + \delta\tilde{M}^2_{ij}, \tilde{B}_{ij} + \delta\tilde{B}_{ij}, \\
\tilde{A}_{ijk}, \tilde{M}_\lambda\} & & \quad \tilde{A}_{ijk} + \delta\tilde{A}_{ijk}, \tilde{M}_\lambda + \delta\tilde{M}_\lambda\} \\
\{u_{3ij}(0), v_3(0)\} & \to & \{u_{3ij}(0) + \delta u_{3ij}(0), v_3(0) + \delta v_3(0)\}
\end{array}
\tag{73}
$$

with real constants $\sqrt{Z_A}$, $\sqrt{Z_\lambda}$, $\sqrt{Z_c}$, $\sqrt{Z_{\phi ij}}$, $\sqrt{Z_{\psi ij}}$, δg, δm_{ij}, δg_{ijk}, $\delta\tilde{M}^2_{ij}$, $\delta\tilde{B}_{ij}$, $\delta\tilde{A}_{ijk}$, $\delta\tilde{M}_\lambda$, $\delta u_{3ij}(0)$, $\delta v_3(0)$ that have to be compatible with the global symmetries.

Similarly, let δR be the following infinitesimal renormalization transformation:

$$
\begin{aligned}
\delta R = \frac{1}{2}\delta Z_A\Big[\int d^4x\Big(& A^\mu_a\frac{\delta}{\delta A^\mu_a} - Y_{A^\mu_a}\frac{\delta}{\delta Y_{A^\mu_a}} \\
& - B_a\frac{\delta}{\delta B_a} - \bar{c}_a\frac{\delta}{\delta\bar{c}_a}\Big) + 2\xi\frac{\partial}{\partial\xi}\Big] \\
+ \frac{1}{2}\delta Z_\lambda \int d^4x\Big(& \lambda_a\frac{\delta}{\delta\lambda_a} + \overline{\lambda}_a\frac{\delta}{\delta\overline{\lambda}_a} \\
& - Y_{\lambda_a}\frac{\delta}{\delta Y_{\lambda_a}} - Y_{\overline{\lambda}_a}\frac{\delta}{\delta Y_{\overline{\lambda}_a}}\Big) \\
+ \frac{1}{2}\delta Z_c \int d^4x\Big(& c\frac{\delta}{\delta c} - Y_c\frac{\delta}{\delta Y_c}\Big) \\
+ \frac{1}{2}\delta Z_{\phi ij} \int d^4x\Big(& \phi_j\frac{\delta}{\delta\phi_i} + \phi^\dagger_j\frac{\delta}{\delta\phi^\dagger_i} \\
& - Y_{\phi_i}\frac{\delta}{\delta Y_{\phi_j}} - Y_{\phi^\dagger_i}\frac{\delta}{\delta Y_{\phi^\dagger_j}}\Big) \\
+ \frac{1}{2}\delta Z_{\psi ij} \int d^4x\Big(& \psi^\alpha_j\frac{\delta}{\delta\psi^\alpha_i} + \overline{\psi}_{j\dot\alpha}\frac{\delta}{\delta\overline{\psi}_{i\dot\alpha}} \\
& - Y^\alpha_{\psi_i}\frac{\delta}{\delta Y^\alpha_{\psi_j}} - Y_{\overline{\psi}_i\dot\alpha}\frac{\delta}{\delta Y_{\overline{\psi}_j\dot\alpha}}\Big) \\
+ \delta g\frac{\partial}{\partial g} + \delta m_{ij}&\frac{\partial}{\partial m_{ij}} + \delta g_{ijk}\frac{\partial}{\partial g_{ijk}}
\end{aligned}
$$

$$+\,\delta\tilde{M}^2_{ij}\frac{\partial}{\partial\tilde{M}^2_{ij}}+\delta\tilde{B}_{ij}\frac{\partial}{\partial\tilde{B}_{ij}}+\delta\tilde{A}_{ijk}\frac{\partial}{\partial\tilde{A}_{ijk}}+\delta\tilde{M}_\lambda\frac{\partial}{\partial\tilde{M}_\lambda}$$
$$+\,\delta u_{3ij}(0)\frac{\partial}{\partial u_{3ij}(0)}+\delta v_3(0)\frac{\partial}{\partial v_3(0)}\ . \tag{74}$$

According to the results of sec. 4.1 and using the identification

$$\begin{aligned}\sqrt{Z_{\phi ij}} &\to u_{1ij}\ ,\\ \sqrt{Z_{\psi ij}} &\to (u_1u_2)_{ij}\ ,\\ \sqrt{Z_A} &\to v_1\ ,\\ \sqrt{Z_\lambda} &\to v_1v_2\ ,\end{aligned}\tag{75}$$

we see that both operators $R, \delta R$ are compatible with the symmetries. Suppose, Γ_{cl} is a classical solution of $\text{Sym}(\Gamma_{\text{cl}})=0$. Then $R\Gamma_{\text{cl}}$ is another solution:

$$\text{Sym}(R\Gamma_{\text{cl}})\ =\ 0\ , \tag{76}$$

and δR generates symmetric counterterms (compare eq. (54)):

$$\begin{aligned}\Gamma_{\text{sym}} &= \delta R\Gamma_{\text{cl}}\\ \Rightarrow\quad \text{Sym}(\Gamma_{\text{cl}}+\zeta\Gamma_{\text{sym}}) &= 0+\mathcal{O}(\zeta^2)\ .\end{aligned}\tag{77}$$

Now we consider the symmetry identities and its classical solutions in the limit (68).

Lemma: Let Γ_{cl} and Γ_{sym} denote a classical solution and an action for symmetric counterterms in the limit $a=\chi=0$,

$$\text{Sym}(\Gamma_{\text{cl}})|_{a=\chi=0}\ =\ 0\ , \tag{78}$$
$$\text{Sym}(\Gamma_{\text{cl}}+\zeta\Gamma_{\text{sym}})|_{a=\chi=0}\ =\ 0+\mathcal{O}(\zeta^2)\ . \tag{79}$$

Then the most general form of Γ_{cl}, Γ_{sym} has to fulfil the relations

$$\Gamma_{\text{cl}}|_{a=\chi=0}\ =\ [R\Gamma_{\text{cl, canonical}}]|_{a=\chi=0}\ , \tag{80}$$
$$\Gamma_{\text{sym}}|_{a=\chi=0}\ =\ [\delta R\Gamma_{\text{cl, canonical}}]|_{a=\chi=0}\ , \tag{81}$$

with the operators R, δR defined in (73), (74).

Proof: The general classical solution of the symmetry identities (78), (79) can be obtained by a straightforward calculation. We write down a general ansatz, apply the symmetry identities and derive the necessary relations the coefficients in the ansatz have to satisfy. Although the calculation is lengthy, the announced results (80), (81) follow in a direct way.

We now give a short sketch of the calculation with emphasis on the main point, namely the restriction of the terms of $\mathcal{O}(\hat{f},\hat{f}^\dagger)$. This sketch will also show why we have to use the limit (68) instead of (16) in the statement of the lemma.

The most general ansatz for Γ_{cl} can be decomposed according to the degree in $a,\chi,\hat{f}$:

$$\Gamma_{\text{cl}}\ =\ \Gamma_0+\Gamma_{\hat{f},\,\text{lin}}+\Gamma_{\hat{f},\,\text{rest}}+\Gamma_{\chi,\,\text{lin}}+\Gamma_{\text{rest}}\ , \tag{82}$$

where Γ_0 does not depend on $a, \chi, \hat{f}$; $\Gamma_{\hat{f},\mathrm{lin}}, \Gamma_{\hat{f},\mathrm{rest}}$ are linear and of higher degree in $\hat{f}$ but do not depend on a, χ; $\Gamma_{\chi,\mathrm{lin}}$ is linear in χ and does not depend on a, $\hat{f}$, and Γ_{rest} contains the rest of the dependence on χ, $\hat{f}$, and the complete dependence on a.

Since all defining symmetry identities either do not change the degree in $a, \chi, \hat{f}$ or increase it, we obtain for Γ_0:

$$0 = \mathrm{Sym}(\Gamma)|_{a=\chi=\hat{f}=0} = \mathrm{Sym}(\Gamma_0) \ , \tag{83}$$

thus Γ_0 is a classical solution of the defining symmetry identities in the case without soft breaking [10].

Next, the symmetry identities in (78) imply that $\Gamma_{\hat{f},\,\mathrm{lin}}$ is globally invariant and does not depend on B_a and ω^μ, and that

$$\begin{aligned} 0 &= S(\Gamma)|_{a=\chi=0,\,\text{linear in } \hat{f}} \\ &= s^0_{\Gamma_0}\,\Gamma_{\hat{f},\mathrm{lin}} + S_\chi(\Gamma_{\chi,\mathrm{lin}}) \ . \end{aligned} \tag{84}$$

Here $s^0_{\Gamma_0}$ is the linearized version of S_0 defined by

$$S_0(\Gamma_0 + \zeta\Gamma_1) = S_0(\Gamma_0) + \zeta s^0_{\Gamma_0}\Gamma_1 + \mathcal{O}(\zeta^2) \ , \tag{85}$$

and

$$\begin{aligned} S_\chi(\Gamma) &= \int d^4x \Big(s\chi^\alpha \frac{\delta\Gamma}{\delta\chi^\alpha}\Big|_{a=\chi=0} + s\overline{\chi}_{\dot\alpha} \frac{\delta\Gamma}{\delta\overline{\chi}_{\dot\alpha}}\Big|_{a=\chi=0} \Big) \\ &= \int d^4x \Big(\sqrt{2}\hat{f}\epsilon^\alpha \frac{\delta\Gamma}{\delta\chi^\alpha}\Big|_{a=\chi=0} \\ &\qquad - \sqrt{2}\hat{f}^\dagger\overline{\epsilon}_{\dot\alpha} \frac{\delta\Gamma}{\delta\overline{\chi}_{\dot\alpha}}\Big|_{a=\chi=0} \Big) \ . \end{aligned} \tag{86}$$

Due to the form of the operator S_χ we obtain

$$s^0_{\Gamma_0}\,\Gamma_{\hat{f},\mathrm{lin}} = \mathcal{O}(\epsilon\hat{f}) + \mathcal{O}(\overline{\epsilon}\hat{f}^\dagger) \ . \tag{87}$$

Since on the physical fields $s^0_{\Gamma_0}$ acts as the BRS operator s up to field and parameter renormalizations, it is easy to see that the most general solution for $\Gamma_{\hat{f},\mathrm{lin}}$ that is compatible with the requirements of sec. 2.3 is given by

$$\begin{aligned} \Gamma_{\hat{f},\mathrm{lin}} &= \hat{f}\Big(\tilde{A}_{ijk}\phi_i\phi_j\phi_k + \tilde{B}_{ij}\phi_i\phi_j + \tilde{M}_\lambda\lambda_a\lambda_a \\ &\qquad + u_{3ij}\sqrt{2}Y_{\psi_i}\epsilon\hat{f}\phi_j + v_3\sqrt{2}Y_{\overline{\lambda}_a}\overline{\sigma}^\mu\epsilon A_{a\mu}\Big) \\ &\qquad + h.c. \end{aligned} \tag{88}$$

All these terms are accounted for in the operator R, eq. (73).

This is the point where the limit (68) is important. If we had required only $\mathrm{Sym}(\Gamma_{\mathrm{cl}})|_{a=\chi=f=0}$ instead of eq. (78), then we would have obtained only $\mathcal{O}(\epsilon) + \mathcal{O}(\overline{\epsilon})$ on the r.h.s. of eq. (87), and in the solution to this equation non-GG terms $\phi\phi\phi^\dagger$ or $\psi\psi$ would have appeared.

The constraints on the remaining parts of Γ_{cl} can be worked out similarly.

5.2 Physical parameters

Once the symmetry identities are satisfied at a given order in the limit (68), there is still the possibility of having divergent contributions and adding symmetric counterterms. The finite parts of these counterterms and equivalently the physical meaning of the parameters have to be fixed by suitable normalization conditions.

Both the divergences and the counterterms have to be local and power-counting renormalizable functionals $\Gamma_{\rm sym}$ satisfying

$$\mathrm{Sym}(\Gamma_{\rm cl}+\zeta\Gamma_{\rm sym})|_{a=\chi=0} \;=\; 0+\mathcal{O}(\zeta^2)\ . \tag{89}$$

According to the lemma the most general form of $\Gamma_{\rm sym}$ is given by the infinitesimal renormalization transformation

$$\Gamma_{\rm sym}|_{a=\chi=0} \;=\; [\delta R\Gamma_{\rm cl}]|_{a=\chi=0}\ . \tag{90}$$

This leads to the following hierarchy of the symmetric counterterms:

1. Counterterms appearing in physical processes, where not only $a=\chi=0$, but also the external Y_i fields are set to zero:

   $$\Gamma_{\rm sym}|_{a=\chi=0,Y_i=0}\ . \tag{91}$$

   This first class contains the counterterms to the field renormalization constants $Z_A, Z_\lambda, Z_c, Z_\phi, Z_\psi$ and the parameters g, m_{ij}, g_{ijk}, $\tilde{M}^2_{ij}$, $\tilde{B}_{ij}$, $\tilde{A}_{ijk}$, $\tilde{M}_\lambda$.

2. Additional counterterms appearing for $Y_i\neq 0$:

   $$\Gamma_{\rm sym}|_{a=\chi=0,Y_i\neq0}\ . \tag{92}$$

   This class contains precisely the counterterms to the u_3, v_3 parameters.

3. The rest of the counterterms appearing for a,χ arbitrary:

   $$\Gamma_{\rm sym}|_{a,\chi\neq0,Y_i\neq0}\ . \tag{93}$$

   This class contains infinitely many independent counterterms.

The normalization conditions fixing the first, second and third set of counterterms we call *normalization conditions of the first, second and third class*, respectively.

The next theorem states how far we get using only the class-one-normalization conditions and leaving open the ones of the second and third class.

Theorem 1: Two solutions Γ_1 and Γ_2 of the same class-one-normalization conditions and of the symmetry identities in the limit (68),

$$\mathrm{Sym}(\Gamma_2)=\mathrm{Sym}(\Gamma_1) \;=\; 0\ , \tag{94}$$

can differ at most by local terms proportional to Y_ψ, Y_λ:

$$\begin{aligned}
&(\Gamma_2-\Gamma_1)|_{a=\chi=0}\\
&\quad= \Delta_Y(u_{3ij}(0)+\delta u_{3ij}(0), v_3(0)+\delta v_3(0))\\
&\quad\equiv \int d^4x\Big(-Y_{\psi_i\alpha}\sqrt{2}\epsilon^\alpha\hat{f}(u_{3ij}(0)+\delta u_{3ij}(0))\phi_j\\
&\qquad\quad - Y_{\lambda\alpha}\sqrt{2}(v_3(0)+\delta v_3(0))\hat{f}^\dagger\overline{\epsilon\sigma}^{\mu\alpha}A_\mu\Big)+h.c.
\end{aligned}\tag{95}$$

Proof: Due to the form of the general classical solution in the limit (68) this holds at the tree level. To perform an inductive proof of this statement we suppose that we have at the order $\hbar^{n-1}$:

$$\begin{aligned}(\Gamma_2 - \Gamma_1)|_{a=\chi=0} &= \Delta_Y(u_3^{(n-1)}, v_3^{(n-1)}) \\ &\quad + \mathcal{O}(\hbar^n)\ , \end{aligned} \tag{96}$$

$$\begin{aligned}(\Gamma_{2,\mathrm{ct}} - \Gamma_{1,\mathrm{ct}})|_{a=\chi=0} &= \Delta_Y(\delta u_3^{(n-1)}, \delta v_3^{(n-1)}) \\ &\quad + \mathcal{O}(\hbar^n)\ . \end{aligned} \tag{97}$$

Then, at the next order *all* one-particle irreducible loop diagrams not involving a, χ are the same, regardless whether calculated according to the Feynman rules for Γ_1 or Γ_2. This is true because even though the Feynman rules differ by the terms Δ_Y, these differences cannot contribute since they are linear in the propagating fields.

The difficult point is to prove that the counterterms of the order $\hbar^n$, denoted by $\Gamma^{(n)}_{1,\ \mathrm{ct}}$ and $\Gamma^{(n)}_{2,\ \mathrm{ct}}$, do not invalidate (96-97). We know

$$\begin{aligned}(\Gamma_2 - \Gamma_1)|_{a=\chi=0} &= \Delta\Gamma^{(n)}_{\mathrm{ct}} + \Delta_Y(u_3^{(n-1)}, v_3^{(n-1)}) \\ &\quad + \mathcal{O}(\hbar^{n+1})\ , \end{aligned} \tag{98}$$

$$\Delta\Gamma^{(n)}_{\mathrm{ct}} = (\Gamma^{(n)}_{2,\ \mathrm{ct}} - \Gamma^{(n)}_{1,\ \mathrm{ct}})|_{a=\chi=0}\ . \tag{99}$$

Thus, taking into account the symmetry of Δ_Y and the fact that all symmetry identities except for the Slavnov-Taylor identity are linear and do not change the degree in a, χ, we obtain for these identities

$$\begin{aligned} 0 &= \mathrm{Sym}(\Gamma_2)|_{a=\chi=0} \\ &= \mathrm{Sym}(\Gamma_2|_{a=\chi=0}) \\ &= \mathrm{Sym}(\Gamma_1|_{a=\chi=0} + \Delta\Gamma^{(n)}_{\mathrm{ct}} \\ &\quad + \Delta_Y(u_3^{(n-1)}, v_3^{(n-1)})) \\ &= 0 + \mathrm{Sym}(\Delta\Gamma^{(n)}_{\mathrm{ct}})\ . \end{aligned} \tag{100}$$

For the Slavnov-Taylor identity we obtain at the order $\hbar^n$ (we use the operator S_χ defined in eq. (86)):

$$\begin{aligned} 0 &= S(\Gamma_2)|_{a=\chi=0} \\ &= S_0(\Gamma_2|_{a=\chi=0}) + S_\chi(\Gamma_2) \\ &= S_0(\Gamma_1|_{a=\chi=0} + \Delta\Gamma^{(n)}_{\mathrm{ct}} + \Delta_Y) + S_\chi(\Gamma_2) \\ &= S(\Gamma_1 + \Delta\Gamma^{(n)}_{\mathrm{ct}} + \Delta_Y)|_{a=\chi=0} \\ &\quad + S_\chi(\Gamma_2 - (\Gamma_1 + \Delta\Gamma^{(n)}_{\mathrm{ct}} + \Delta_Y)) \\ &= S(\Gamma_1 + \Delta\Gamma^{(n)}_{\mathrm{ct}})|_{a=\chi=0} \\ &\quad + \int d^4x \Big(\frac{\delta\Gamma_1 + \Delta\Gamma^{(n)}_{\mathrm{ct}}}{\delta Y_i} \frac{\delta\Delta_Y}{\delta\varphi_i} \\ &\qquad + \frac{\delta\Delta_Y}{\delta Y_i} \frac{\delta\Gamma_1 + \Delta\Gamma^{(n)}_{\mathrm{ct}}}{\delta\varphi_i} \Big)|_{a=\chi=0} \\ &\quad + S_\chi(\Gamma_2 - (\Gamma_1 + \Delta\Gamma^{(n)}_{\mathrm{ct}} + \Delta_Y)) \end{aligned}$$

$$
\begin{aligned}
&= S(\Gamma_1 + \Delta\Gamma_{\mathrm{ct}}^{(n)})|_{a=\chi=0} \\
&\quad + \sqrt{2}(\epsilon^\alpha X_\alpha \hat{f} - \overline{\epsilon}_{\dot\alpha}\bar{X}^{\dot\alpha}\hat{f}^\dagger) \ . \qquad (101)
\end{aligned}
$$

The last two equations hold owing to the special form of Δ_Y with some suitably chosen functional X_α. Since Γ_1 satisfies the Slavnov-Taylor identity the first term of this result can be simplified using

$$
S(\Gamma_1 + \Delta\Gamma_{\mathrm{ct}}^{(n)}) \;=\; S(\Gamma_{1,\mathrm{cl}} + \Delta\Gamma_{\mathrm{ct}}^{(n)}) + \mathcal{O}(\hbar^{n+1}) \ . \qquad (102)
$$

Therefore, both terms in the last line of eq. (101) are local and power-counting renormalizable functionals of the order $\hbar^n$, and we can define a counterterm action

$$
\Gamma_{\mathrm{sym}} \;=\; \Delta\Gamma_{\mathrm{ct}}^{(n)} + (\chi^\alpha X_\alpha + \overline{\chi}_{\dot\alpha}\bar{X}^{\dot\alpha}) \qquad (103)
$$

that satisfies

$$
\begin{aligned}
S(\Gamma_{1,\mathrm{cl}} + \Gamma_{\mathrm{sym}})|_{a=\chi=0} &= S(\Gamma_1 + \Delta\Gamma_{\mathrm{ct}}^{(n)})|_{a=\chi=0} \\
&\quad + \sqrt{2}(\epsilon^\alpha X_\alpha \hat{f} - \overline{\epsilon}_{\dot\alpha}\bar{X}^{\dot\alpha}\hat{f}^\dagger) \\
&= 0 + \mathcal{O}(\hbar^{n+1}) \ . \qquad (104)
\end{aligned}
$$

Thus, Γ_{sym} is a symmetric counterterm in the sense of eq. (89), and we obtain from the lemma:

$$
\Gamma_{\mathrm{sym}}|_{a=\chi=0} \;=\; [\delta R \Gamma_{1,\mathrm{cl}}]|_{a=\chi=0} \qquad (105)
$$

On the other hand, by construction Γ_{sym} contains the relevant difference of Γ_1 and Γ_2 at the order $\hbar^n$:

$$
\begin{aligned}
(\Gamma_2 - \Gamma_1)|_{a=\chi=0} &= \Gamma_{\mathrm{sym}}|_{a=\chi=0} + \Delta_Y(u_3^{(n-1)}, v_3^{(n-1)}) \\
&\quad + \mathcal{O}(\hbar^{n+1}) \ . \qquad (106)
\end{aligned}
$$

Now, since $\Gamma_{1,2}$ satisfy the same class-one-normalization conditions, Γ_{sym} cannot contain any class-one-counterterms. Since these are the only counterterms that appear in the limit $a = \chi = Y_i = 0$, we obtain

$$
\Gamma_{\mathrm{sym}}|_{a=\chi=Y_i=0} \;=\; 0 \ . \qquad (107)
$$

Owing to the concrete form of δR, this shows

$$
\Delta\Gamma_{\mathrm{ct}}^{(n)}|_{a=\chi=0} \;=\; \Gamma_{\mathrm{sym}}|_{a=\chi=0} = \Delta_Y(\delta u_3^{(n)}, \delta v_3^{(n)}) \ . \qquad (108)
$$

Together with eq. (98) this demonstrates the validity of eqs. (96-97) at the next order, completing the induction.

5.3 Simplified symmetry identities at the quantum level

While according to theorem 1 only a finite number of parameters is physically relevant, theorem 2 states that it is sufficient to establish the symmetry identities in the limit (68), where the unphysical parameters do not play a role. This implies that the unphysical parameters do not even appear in practice.

A generalization of theorem 2 is proven at the classical level by the Lemma in subsec. 5.1 together with eqs. (76), (77). In this subsection the theorem is extended to the quantum level. The statement of the theorem and its proof at the quantum level is divided into two parts—the existence of a solution to the symmetry identities in the limit (68) and its extension to a full solution.

5.3.1 Existence of a solution

Theorem 2a: Suppose Γ is a solution of the symmetry identities in the limit (68) up to the order $\hbar^{n-1}$,

$$\mathrm{Sym}(\Gamma)|_{a=\chi=0} = 0 + \mathcal{O}(\hbar^n) , \tag{109}$$

and Γ^{exact} is an extension that solves the full symmetry identities,

$$\mathrm{Sym}(\Gamma^{\mathrm{exact}}) = 0 + \mathcal{O}(\hbar^n) , \tag{110}$$

$$(\Gamma^{\mathrm{exact}} - \Gamma)|_{a=\chi=0} = 0 + \mathcal{O}(\hbar^n) . \tag{111}$$

Then we claim that Γ, Γ^{exact} can be renormalized in such a way that the eqs. (109-111) are maintained at the next order $\hbar^n$.

Proof: Since we assume the absence of anomalies, Γ^{exact} can be renormalized in such a way that

$$\mathrm{Sym}(\Gamma^{\mathrm{exact}}) = 0 + \mathcal{O}(\hbar^{n+1}) . \tag{112}$$

Since the Feynman rules of the order $\hbar^n$ defined by Γ^{exact} and Γ differ only in terms $\sim a, \chi$, all loop diagrams contributing to $\Gamma^{\mathrm{exact}}|_{a=\chi=0}$ and $\Gamma|_{a=\chi=0}$ are equal at this order. Thus, adding appropriate $\mathcal{O}(\hbar^n)$ counterterms to Γ we obtain

$$(\Gamma^{\mathrm{exact}} - \Gamma)|_{a=\chi=0} = 0 + \mathcal{O}(\hbar^{n+1}) . \tag{113}$$

However, Γ does not yet satisfy the Slavnov-Taylor identity at this order. Indeed, neglecting terms of the order $\hbar^{n+1}$ we obtain

$$\begin{aligned} S(\Gamma)|_{a=\chi=0} &= S_0(\Gamma|_{a=\chi=0}) + S_\chi(\Gamma) \\ &= S_0(\Gamma^{\mathrm{exact}}|_{a=\chi=0}) + S_\chi(\Gamma) \\ &= S(\Gamma^{\mathrm{exact}})|_{a=\chi=0} + S_\chi(\Gamma - \Gamma^{\mathrm{exact}}) \\ &= S_\chi(\Gamma - \Gamma^{\mathrm{exact}}) \\ &= \hbar^n \Delta . \end{aligned} \tag{114}$$

Owing to the form of S_χ and to the quantum action principle [19], the lowest order of Δ is a local and power-counting renormalizable functional of the form

$$\Delta = \int \sqrt{2}\epsilon^\alpha X_\alpha \hat{f} - \sqrt{2}\bar{\epsilon}_{\dot\alpha} \bar{X}^{\dot\alpha} \hat{f}^\dagger + \mathcal{O}(\hbar) . \tag{115}$$

Hence, adding the counterterms

$$\Gamma \to \Gamma - \int \hbar^n (\chi^\alpha X_\alpha + \overline{\chi}_{\dot\alpha} \bar{X}^{\dot\alpha}) \tag{116}$$

restores the Slavnov-Taylor identity without interfering with eq. (113). All further symmetry identities are linear and homogeneous in a, χ. Therefore, Γ satisfies these identities, too, and we obtain

$$\mathrm{Sym}(\Gamma)|_{a=\chi=0} = 0 + \mathcal{O}(\hbar^{n+1}) . \tag{117}$$

This was to be shown.

5.3.2 Extension to a full solution

Theorem 2b: Let Γ be a solution to the symmetry identities in the limit $a = \chi = 0$,

$$\mathrm{Sym}(\Gamma)|_{a=\chi=0} \quad = \quad 0\ . \tag{118}$$

Then there exists an extension to a full solution Γ^{exact} satisfying

$$\begin{aligned} \mathrm{Sym}(\Gamma^{\mathrm{exact}}) &= 0\ , && (119)\\ (\Gamma^{\mathrm{exact}} - \Gamma)|_{a=\chi=0} &= 0\ . && (120) \end{aligned}$$

Proof: Due to the lemma there is a classical solution $\Gamma^{\mathrm{exact}}_{\mathrm{cl}}$ satisfying eqs. (119-120). Now suppose the same is true at the order $\hbar^{n-1}$, that is there exists an effective action Γ^{exact} satisfying

$$\begin{aligned} \mathrm{Sym}(\Gamma^{\mathrm{exact}}) &= 0 + \mathcal{O}(\hbar^n)\ , && (121)\\ (\Gamma^{\mathrm{exact}} - \Gamma)|_{a=\chi=0} &= 0 + \mathcal{O}(\hbar^n)\ . && (122) \end{aligned}$$

Then, according to theorem 2a there are $\mathcal{O}(\hbar^n)$ counterterms yielding $\widetilde{\Gamma} = \Gamma + \mathcal{O}(\hbar^n)$, $\widetilde{\Gamma}^{\mathrm{exact}} = \Gamma^{\mathrm{exact}} + \mathcal{O}(\hbar^n)$ such that

$$\begin{aligned} \mathrm{Sym}(\widetilde{\Gamma})|_{a=\chi=0} &= 0 + \mathcal{O}(\hbar^{n+1})\ , && (123)\\ \mathrm{Sym}(\widetilde{\Gamma}^{\mathrm{exact}}) &= 0 + \mathcal{O}(\hbar^{n+1})\ , && (124)\\ (\widetilde{\Gamma}^{\mathrm{exact}} - \widetilde{\Gamma})|_{a=\chi=0} &= 0 + \mathcal{O}(\hbar^{n+1})\ . && (125) \end{aligned}$$

However, due to eqs. (118), (123) the difference $\widetilde{\Gamma} - \Gamma$ has to be a symmetric counterterm as defined in eq. (79). Hence, it has the form

$$(\Gamma - \widetilde{\Gamma})|_{a=\chi=0} \quad = \quad [\delta R \Gamma_{\mathrm{cl}}]|_{a=\chi=0}\ . \tag{126}$$

Therefore, $\Gamma^{\mathrm{exact}} = \widetilde{\Gamma}^{\mathrm{exact}} + \delta R \Gamma^{\mathrm{exact}}_{\mathrm{cl}}$ has the desired properties

$$\begin{aligned} \mathrm{Sym}(\Gamma^{\mathrm{exact}}) &= \mathrm{Sym}(\widetilde{\Gamma}^{\mathrm{exact}} + \delta R \Gamma^{\mathrm{exact}}_{\mathrm{cl}})\\ &= 0 + \mathcal{O}(\hbar^{n+1})\ , && (127)\\ (\Gamma^{\mathrm{exact}} - \Gamma)|_{a=\chi=0} &= (\widetilde{\Gamma}^{\mathrm{exact}} - \widetilde{\Gamma})|_{a=\chi=0}\\ &= 0 + \mathcal{O}(\hbar^{n+1})\ . && (128) \end{aligned}$$

This completes the induction.

6 Alternative approaches

6.1 Alternative Slavnov-Taylor identity for soft breaking

A Slavnov-Taylor identity describing supersymmetric Yang-Mills theories with soft breaking in the Wess-Zumino gauge has already been introduced in ref. [7]. Basically, as in our construction the soft breaking is introduced via external fields with definite BRS transformation rules. These transformation rules contain a constant shift that yields the soft parameters in the limit of vanishing external fields. But the detailed structure of the construction in [7] is different from ours, and a priori it could be that at the quantum level both constructions describe two different theories, although in the classical limit they reproduce the same soft breaking terms.

The main difference concerns the underlying intuition and consequently the external field content:[6] The soft breaking terms are not introduced as couplings to a multiplet $(a, \chi, \hat{f})$ that transforms as a chiral supermultiplet but as couplings to a BRS doublet $(u, \hat{v})$ where[7]

$$
\begin{aligned}
su &= \hat{v} - i\omega^\nu \partial_\nu u \,, && (129)\\
sv &= 2i\epsilon\sigma^\nu \bar{\epsilon}\partial_\nu u - i\omega^\nu \partial_\nu v \,, && (130)\\
\hat{v}(x) &= v(x) + \kappa \,. && (131)
\end{aligned}
$$

The main benefit of this structure is that the cohomological sector of the theory is not altered compared to the case without soft breaking. This allows a straightforward proof of the absence of anomalies. Contrary to the case of $(a, \chi, \hat{f})$, however, the BRS transformations of u and v cannot be interpreted as supersymmetry transformations where simply the transformation parameter has been promoted to a ghost. Moreover, u and v are two scalar fields and therefore cannot form a supersymmetry multiplet.

In the limit of vanishing external fields the classical action in both approaches reduces to the same soft breaking action but for non-vanishing external fields in both cases new parameters appear: in our case infinitely many, as discussed in section 4.1, in the case of [7] finitely many, including for instance the parameters ρ_2, ρ_4 that appear in the terms

$$
\begin{aligned}
\Gamma_{2,4} \;=\; & \int d^4x \Big(\rho_{2ab} Y_{\psi_b \alpha} \epsilon^\alpha (\hat{v}\phi_a - \sqrt{2} u \epsilon \psi_a) \\
& + \rho_{4ab} \hat{v}\bar{u}\epsilon\psi_a \phi_b^\dagger \Big) + \ldots && (132)
\end{aligned}
$$

The main reason why the approach of ref. [7] cannot be used directly in phenomenological studies is that the physical meaning of these parameters is not discussed. In particular, a theorem showing whether these parameters are irrelevant for physical quantities or not—analogous to sec. 5.2—is lacking.

In spite of these differences, there is a remarkable relation between both approaches. First of all, the quantum numbers of $\hat{v}$ and $\hat{f}$ are equal, and second we can combine the supersymmetry ghost and u to a spinor (ϵu) that has the same quantum numbers as χ. Hence, we can identify

$$
\begin{aligned}
a &\to 0 \,, \\
\chi^\alpha &\to \epsilon^\alpha u \,, \\
\sqrt{2}\hat{f} &\to \hat{v} \,. && (133)
\end{aligned}
$$

Furthermore, this correspondence even holds for the BRS transformations:

$$
\begin{aligned}
sa &\to \sqrt{2}\epsilon\epsilon u = 0 \,, \\
s\chi^\alpha &\to \sqrt{2}\epsilon^\alpha \hat{v} - i\omega^\nu \partial_\nu \epsilon^\alpha u = s\epsilon^\alpha u \,, \\
s\sqrt{2}\hat{f} &\to 2i\bar{\epsilon}\sigma^\nu \partial_\nu \epsilon u - i\omega^\nu \partial_\nu \hat{v} = s\hat{v} \,. && (134)
\end{aligned}
$$

[6]One further difference concerns the supersymmetric mass terms which are also introduced via external fields in [7]. This is done in order not to violate R-invariance because the R-weights of the chiral fields are fixed to $n_i = \frac{2}{3}$ (translated to our convention) in accordance with the R-part of the supercurrent. In our case the R-weights are assumed to be chosen in such a way that the mass terms are invariant and therefore we do not need such an external field multiplet.

[7]We translate to our conventions. In particular, in [7] there is also an R-transformation part in the BRS transformations, which we neglect here.

Here we have used $\epsilon^\alpha\epsilon_\alpha = 0$, which holds since ϵ is bosonic. Thus, u and $\hat{v}$ may be regarded as a part of our chiral multiplet $(a, \chi, \hat{f})$. And there is a natural identification in our framework of terms like the ρ_2-term in (132), where u comes always in combination with ϵ. In fact, this term has the same structure as the u_3-term in eq. (57) with $u_3 \to -\rho_2$ when (133) is used.

However, in the classical action of [7] there are also terms where u appears without an accompanying ϵ or $\bar{u}$ without accompanying $\overline{\epsilon}$, such as the ρ_4-term in (132). These terms have no correspondence in our framework. On the other hand, of course our terms depending on the a field have no correspondence in [7]. Therefore both frameworks are really different, and none is just a stronger or weaker version of the other.

Still, we can formulate the following statement, which can be checked trivially using (134):

Suppose, $\Gamma(a, \chi, \hat{f}, \dots)$ is a solution of our symmetry identities to all orders. Then

$$\begin{aligned}&\Gamma_{MPW}(u, \hat{v}, \dots)\\ &\quad= \Gamma(a = 0, \chi = \epsilon u, \hat{f} = \hat{v}/\sqrt{2}, \dots)\end{aligned} \tag{135}$$

is a solution to the symmetry identities of [7] to all orders.

Thus, every solution in our framework generates a special solution of the framework of [7] that depends on u only via $\chi = \epsilon u$. From this we can draw two conclusions: First, the parameters like ρ_4 can be set to zero consistently to all orders. Second, in this case we can apply our statement about the physically relevant parameters proven in section 5.2 also on Γ_{MPW}, so the physically relevant part of Γ_{MPW} does not depend on parameters like ρ_2.

6.2 Additional soft breaking terms

As discussed in section 2.2, in many concrete models the soft breaking terms of the GG class are not the only ones that produce no quadratic divergences [12]. The additional breaking terms are scalar interactions of the type[8]

$$h_{ijk}\phi_i\phi_j\phi_k^\dagger \ . \tag{136}$$

These terms cannot appear in a classical solution to our symmetry identities for the simple reason that otherwise in $S(\Gamma_{\rm cl})|_{a=\chi=0}$ terms such as

$$\begin{aligned}&s\int d^4x\ h_{ijk}\phi_i\phi_j\phi_k^\dagger + S_\chi(\Gamma_{\rm cl})\\ &\quad= \int d^4x\sqrt{2}(h_{ijk}+h_{jik})\epsilon\psi_i\phi_j\phi_k^\dagger\\ &\qquad+\sqrt{2}h_{ijk}\phi_i\phi_j\overline{\psi}_k\overline{\epsilon} + \mathcal{O}(\epsilon\hat{f}) + \mathcal{O}(\overline{\epsilon}\hat{f}^\dagger)\end{aligned} \tag{137}$$

would appear that cannot cancel for any possible choice of $h_{ijk} = h_{ijk}(\hat{f}, \hat{f}^\dagger)$. This generic feature of our approach would apparently not be affected by additional η-multiplets with modified quantum numbers such as the R-weight. The only way to allow such terms in our formalism would be to abandon the assumptions made in section 2.3 and allow a mixing between scalar and hermitian conjugate scalar fields

$$\phi_k^\dagger \leftrightarrow \phi_l \ . \tag{138}$$

[8]Further mass terms for chiral fermions can be absorbed in the superpotential.

In this case terms such as

$$\Gamma_{\text{ext}} = \ldots + \int d^4x Z_{kl} Y_{\phi_l} \sqrt{2}\overline{\psi}_k \overline{\epsilon} \ , \tag{139}$$

$$\Gamma_{\text{kin}} = \ldots + \int d^4x Z_{kl} (D^\mu \phi_k)(D_\mu \phi_l) \tag{140}$$

could show up in the classical action. The first of these terms would modify the supersymmetry transformations in such a way that (136) could appear in the supersymmetric part of the action, but at the same time the second term, an unwanted mixing kinetic term, could also not be forbidden.

However, additional soft breaking terms can be naturally introduced by a slight change of the formalism of [7]. There R-invariance is a part of the BRS transformations, but it is also required globally for the complete action, which allows terms of the form suM in the action only if the combination uM has zero R-weight. If we give up the requirement of global R-invariance, then *all* supersymmetry and R-invariance breaking terms of dimension ≤ 3 are possible, in particular (136) can be generated from

$$su\phi_i\phi_j\phi_k^\dagger = \kappa\phi_i\phi_j\phi_k^\dagger + \ldots \ , \tag{141}$$

but terms of dimension 4 like (139), (140) that break R-invariance are excluded. Hence, in this way it can be shown that supersymmetric models with all possible breaking terms of dimension 3 are renormalizable as well as those containing only GG soft breaking terms.

7 Renormalization of supersymmetric QCD

7.1 The model

As an example and a useful application we work out a renormalization scheme for supersymmetric QCD (SQCD). Particular attention is payed to the renormalization of the squark mixing. Since QCD does not contain flavour changing transitions we restrict ourselves to one quark species. Thus, the model contains one $SU(3)$ vector multiplet containing the gluons G_a^μ and gluinos

$$\tilde{g}_a = \begin{pmatrix} -i\lambda_{a\alpha} i\overline{\lambda}_a^{\dot\alpha} \end{pmatrix} \tag{142}$$

with the Weyl spinors λ, $\overline{\lambda}$, and two chiral multiplets ($\tilde{q}_L$,ψ_L), ($\tilde{q}_R^\dagger$, ψ_L^c) in the $SU(3)$ representations 3 and 3^*, respectively. The ψ_L, ψ_L^c spinors combine to the quark Dirac spinor q as

$$q = \begin{pmatrix} \psi_{L\alpha} \overline{\psi}_L^{c\,\dot\alpha} \end{pmatrix} \ . \tag{143}$$

There is only one gauge-invariant superpotential term[9]

$$W = m\tilde{q}_R^\dagger \tilde{q}_L \ . \tag{144}$$

[9]The conventional notation for the scalar component of the second chiral multiplet hides the holomorphic nature of the superpotential.

The most general gauge-invariant soft breaking terms are[10]

$$\Gamma_{\rm soft} = \int d^4x \Big[-\frac{1}{2}\tilde{m}_{\tilde{g}}\overline{\tilde{g}}_a(P_L\hat{f} + P_R\hat{f}^\dagger)\tilde{g}_a \\ - \begin{pmatrix} \tilde{q}_L^\dagger & \tilde{q}_R^\dagger \end{pmatrix} \begin{pmatrix} |\hat{f}|^2\tilde{M}_L^2 & m\hat{f}^\dagger\tilde{M}_{LR} \\ m\hat{f}\tilde{M}_{LR} & |\hat{f}|^2\tilde{M}_R^2 \end{pmatrix} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix} \Big] , \tag{145}$$

where we have retained $f \neq 0$ and the final soft parameters are obtained as in sec. 2.2. Without loss of generality it is possible to assign the R-weights $n_i = 1$ to both chiral multiplets.

Loop calculations for physical processes are done using the following procedure:

1. Calculate the loop graphs using some arbitrary (preferably consistent) regularization. To be definite, we refer to dimensional regularization [21].

2. Establish the symmetry identities
$$\mathrm{Sym}(\Gamma)|_{a=\chi=0} \;=\; 0 \tag{146}$$
using appropriate counterterms if they are violated at the regularized level. This step can be streamlined considerably, and forgetting it can lead to significant numerical errors [11, 22].

3. Add counterterms in order to absorb the remaining divergences. According to sec. 5 only counterterms corresponding to field and parameter renormalization are necessary. Normalization conditions of class one fix the finite part of these counterterms and define the physical meaning of the parameters.

To exemplify the results of sec. 5 we now give a complete list of class-one-normalization conditions for SQCD. The relevant parameters of SQCD are the gauge coupling g, the mass parameter m, and the soft parameters $\tilde{m}_{\tilde{g}}$, $\tilde{M}_L^2$, $\tilde{M}_R^2$, $\tilde{M}_{LR}$. The class-one-normalization conditions have to fix these parameters as well as the field renormalization constants Z_G, Z_c, $Z_{\tilde{g}}$, Z_ψ, Z_{ψ^c}, Z_{ϕ_L}, Z_{ϕ_R}. The QCD part we define in the $\overline{MS}$ scheme:

$$\Gamma_{q\bar{q}G^\mu} \; : \; \overline{MS}, \tag{147}$$
$$\frac{\partial}{\partial p^2}\Gamma^{\rm trans}_{G^\mu G^\nu} \; : \; \overline{MS}, \tag{148}$$
$$\frac{\partial}{\partial p^2}\Gamma_{\bar{c}_a c_b} \; : \; \overline{MS}, \tag{149}$$
$$\Gamma_{q\bar{q}} \; : \; \overline{MS}, \tag{150}$$
$$\frac{\partial}{\partial \not{p}}\Gamma_{q\bar{q}} \; : \; \overline{MS}. \tag{151}$$

This means that at each order the overall counterterms to these quantities absorb the purely divergent terms $\Delta = \frac{2}{4-D} - \gamma + \log 4\pi$ with the space-time dimension D and the Euler-Mascheroni constant γ. Due to non-supersymmetric counterterms added in step 2, this implies that related quantities like $\Gamma_{q\tilde{q}\tilde{g}}$ are not minimally subtracted [20]. The gluino self energy is renormalized in the on-shell scheme:

$$\mathrm{Re}\Gamma_{\tilde{g}\overline{\tilde{g}}}(\not{p} = m_{\tilde{g}}) \;=\; 0 \,, \tag{152}$$
$$\left(\frac{1}{\not{p} - m_{\tilde{g}}}\mathrm{Re}\Gamma_{\tilde{g}\overline{\tilde{g}}}\right)\Bigg|_{\not{p}=m_{\tilde{g}}} \;=\; 1 \,. \tag{153}$$

[10] Due to gauge invariance no additional non-standard soft breaking terms are possible here.

This defines $m_{\tilde{g}}$ to be the physical gluino mass. These conditions determine the counterterms δg, δZ_G, δZ_c, δm, δZ_ψ, δZ_{ψ^c}, $\delta \tilde{m}_{\tilde{g}}$, $\delta Z_{\tilde{g}}$. The conditions for the squark sector are given in the next subsection.

7.2 Renormalization of the squark mixing

For the remaining five symmetric counterterms δZ_{ϕ_L}, δZ_{ϕ_R}, $\delta\tilde{M}_L^2$, $\delta\tilde{M}_R^2$, $\delta\tilde{M}_{LR}$ we can impose five normalization conditions. While sufficient to absorb all divergences according to sec. 5, this is not enough to establish complete on-shell conditions for both squark mass eigenstates. One possible requirement is the following. Choosing some fixed mixing angle $\theta_{\tilde{q}}$ and masses m_1, m_2 we define

$$
\begin{aligned}
(\tilde{q}_1\tilde{q}_2) &= D^T\,(\tilde{q}_L\tilde{q}_R)\ , && (154)\\
D &= \begin{pmatrix} \cos\theta_{\tilde{q}} & \sin\theta_{\tilde{q}} \\ -\sin\theta_{\tilde{q}} & \cos\theta_{\tilde{q}} \end{pmatrix} && (155)
\end{aligned}
$$

and require the following five normalization conditions for the squark self energies $\Gamma^{\tilde{q}}_{ij}$:

$$
\begin{aligned}
\mathrm{Re}\Gamma^{\tilde{q}}_{11}(p^2=m_1^2) &= 0\ , && (156)\\
\frac{\partial}{\partial p^2}\mathrm{Re}\Gamma^{\tilde{q}}_{11}(p^2=m_1^2) &= 1\ , && (157)\\
\mathrm{Re}\Gamma^{\tilde{q}}_{22}(p^2=m_2^2) &= 0\ , && (158)\\
\frac{\partial}{\partial p^2}\mathrm{Re}\Gamma^{\tilde{q}}_{22}(p^2=m_2^2) &= 1\ , && (159)\\
\mathrm{Re}\Gamma^{\tilde{q}}_{12}(p^2=m_1^2) &= 0\ . && (160)
\end{aligned}
$$

These conditions define $\tilde{q}_1$ to be a squark mass eigenstate with mass m_1.[11] However, $\tilde{q}_2$ is not a mass eigenstate and m_2 is not a mass eigenvalue since at the quantum level $\mathrm{Re}\Gamma^{\tilde{q}}_{12}(m_2^2)\neq 0$ and therefore $\det(\mathrm{Re}\Gamma^{\tilde{q}}_{ij}(m_2^2))\neq 0$.

Note that the definition of $\tilde{q}_{1,2}$ is merely a substitution of the R-eigenstate fields $\tilde{q}_{L,R}$ by more physical fields, so it does not introduce new kinds of counterterms into the action. Instead, this substitution leads to a convenient reparametrization of the action and of the symmetric counterterms generated by the renormalization transformations.

7.3 Comparison: renormalization of the squark mixing angle

It might be instructive to compare our scheme, which does not involve a mixing angle counterterm, to the ones of [24], where the first renormalization schemes for the mixing angle have been proposed[12]. In these approaches squark fields $\tilde{q}_{1,2}$ are used that are general linear combinations (non-orthogonal at the quantum level) of $\tilde{q}_{L,R}$. Written in terms of these $\tilde{q}_{1,2}$ fields the Lagrangian has more independent parameters than in our case. A priori, by such a procedure symmetry-breaking counterterms could be generated and normalization conditions could be imposed that contradict the Slavnov-Taylor identity. However, by comparing the schemes of [24] to ours, we can show that they are in accordance with the defining symmetries of the model.

[11] Starting at two-loop order, this definition differs from the one prescribing the real part of the complex pole of the propagators by the physical meaning and the gauge parameter dependence of the mass parameters [23].

[12] Further alternative schemes have been used in [25].

In [24] the following renormalization transformation is performed to obtain the counterterm Lagrangian:

$$
\begin{aligned}
(\tilde{q}_1 \tilde{q}_2) &\to \left(1 + \frac{1}{2}\delta Z\right)(\tilde{q}_1 \tilde{q}_2) \ , && (161)\\
\frac{1}{2}\delta Z &= \begin{pmatrix} \frac{1}{2}\delta Z_{11} & \delta Z_{12} \\ \delta Z_{21} & \frac{1}{2}\delta Z_{22} \end{pmatrix} \ , && (162)\\
\theta_{\tilde{q}} &\to \theta_{\tilde{q}} + \delta\theta_{\tilde{q}} \ , && (163)\\
m_i &\to m_i + \delta m_i, \qquad i = 1, 2 \ . && (164)
\end{aligned}
$$

Thus, in these approaches seven instead of five independent counterterms are present in the squark sector and it is possible to establish

$$
\Gamma^{\tilde{q}}_{12}(p^2 = m_2) \;=\; 0 \qquad (165)
$$

in addition to (156-160). Hence, in this scheme $\tilde{q}_2$ is a squark mass eigenstate and m_2 is the corresponding mass.

The relation of these schemes to ours is the following. Once $\tilde{q}_{1,2}$ are renormalized according to our scheme, we can perform an additional, non-orthogonal substitution

$$
(\tilde{q}_1 \tilde{q}_2)^{\text{new}} \;=\; \begin{pmatrix} 1 & 0 \\ \delta z_1 & 1 + \delta z_2 \end{pmatrix} (\tilde{q}_1 \tilde{q}_2) \qquad (166)
$$

with two UV finite parameters δz_1, δz_2. Since this is only a substitution of variables no symmetries are invalidated. Choosing the parameters δz_1, δz_2 appropriately, the self energies of the new squark fields satisfy the complete on-shell conditions (156-160), (165). The renormalization constants of [24] are simply a convenient reparametrization of ours supplemented by δz_1, δz_2. In this way the schemes of [24] are shown to be correct, and moreover, the finiteness of two of the seven counterterms becomes manifest.

8 Conclusions

In this article we have performed the renormalization of supersymmetric Yang-Mills theories with soft supersymmetry-breaking terms of the GG class. Introducing these terms in a supersymmetric way via an external chiral multiplet, we have constructed a Slavnov-Taylor identity serving as the basic definition of the models. We have derived the correct gauge fixing and ghost terms and proved the cancellation of the divergences.

In the course of our construction, an infinite number of additional parameters appears. However, in sec. 5 it is shown that these parameters are irrelevant. Even better than gauge parameters they do not influence any vertex functions that occur in physical S-matrix elements; and neither at the classical nor at the quantum level it is necessary to calculate the part of the Lagrangian and the counterterms involving those additional parameters.

For practical calculations of physical processes the theorems in sec. 5 imply, first, that the symmetry identities need to be established only in the limit (68), $\mathrm{Sym}(\Gamma)|_{a=\chi=0} = 0$. And second, renormalization of the fields and parameters appearing in the relevant part of the classical action suffices to cancel the divergences.

All results of this article can be easily transferred to the abelian case by adding the external chiral multiplet to the construction in [11]. The essential changes are an additional defining symmetry identity and the relation $Z_A = Z_\lambda = Z_c$ between the field renormalization constants.

Since the supersymmetric extensions of the standard model such as the minimal one (MSSM) involve soft breaking, our results should be an important building block for the renormalization of such models. Indeed, it is straightforward to conclude along similar lines as done here that in the MSSM all ultraviolet divergences can be absorbed by renormalization of the fields and parameters in the classical action. In particular, it is possible to restrict the soft breaking terms to the GG class, as it is usually done. The additional non-GG soft breaking terms described in sec. 2.2 can be included or excluded to obtain a different phenomenology, but they are not necessary in order to render the MSSM renormalizable. However, in the standard model and all its extensions it is of vital importance to solve the special complications that are due to the spontaneous symmetry breaking and the non-semisimple gauge group, in particular the renormalization of the gauge fixing and the mixing between massless particles (photon and its corresponding ghost) and massive ones. For the standard model itself this was done in [8], and for the MSSM, too, these problems deserve a specific treatment.

We have exemplified our results in supersymmetric QCD and given a complete list of physically relevant ("class one") normalization conditions and the counterterms relevant for the squark mixing. While in [24] seven independent counterterms were used we have proven that five are sufficient to cancel all divergences to all orders. Thus, we could derive two finite combinations between the seven counterterms of [24]. Nonetheless, it might seem surprising that different field renormalization constants for the different components of the supermultiplets are necessary: $Z_A \neq Z_\lambda$, $Z_\phi \neq Z_\psi$. However, this necessity may be understood by the appearance of loop corrections to the non-linear BRS transformations. As demonstrated in [11], such loop corrections precisely make up for the different Z factors.

We have discussed GG and non-GG soft breaking terms and compared them at the tree level (sec. 2.2) and at the level of Slavnov-Taylor identities (sec. 6.2). The impossibility to accommodate for non-GG breaking terms in our framework, where breaking terms are introduced via a coupling to a supermultiplet, shows that GG terms are still more consistent with supersymmetry than arbitrary breakings. In contrast, by a slight change of the approach of [7] it is possible to introduce all dimension-3 breakings via couplings to a BRS doublet. It seems therefore quite likely that properties such as non-renormalization theorems that are deeply related to supersymmetry only hold as long as the breaking terms are restricted to the GG class, and that the framework chosen in this paper is a good starting point for a study of these properties.

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A Conventions

2-Spinor indices and scalar products:

$$\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha}, \quad \epsilon_{12} = 1, \quad \epsilon^{\alpha\beta}\epsilon_{\beta\gamma} = \delta^\alpha{}_\gamma, \tag{167}$$

$$\epsilon_{\dot\alpha\dot\beta} = -\epsilon_{\dot\beta\dot\alpha}, \quad \epsilon_{\dot 1\dot 2} = 1, \quad \epsilon^{\dot\alpha\dot\beta}\epsilon_{\dot\beta\dot\gamma} = \delta^{\dot\alpha}{}_{\dot\gamma}, \tag{168}$$

$$\psi\chi = \psi^\alpha\chi_\alpha\ , \quad \psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta\ , \tag{169}$$

$$\overline{\psi}\overline{\chi} = \overline{\psi}_{\dot\alpha}\overline{\chi}^{\dot\alpha}\ , \quad \overline{\psi}_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\overline{\psi}^{\dot\beta}\ . \tag{170}$$

σ **matrices:**

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{171}$$

$$\sigma^\mu_{\alpha\dot\alpha} = (1, \sigma^k)_{\alpha\dot\alpha} \ , \quad \overline{\sigma}^{\mu\dot\alpha\alpha} = (1, -\sigma^k)^{\dot\alpha\alpha} \ , \tag{172}$$

$$(\sigma^{\mu\nu})_\alpha{}^\beta = \frac{i}{2}(\sigma^\mu\overline{\sigma}^\nu - \sigma^\nu\overline{\sigma}^\mu)_\alpha{}^\beta \ , \tag{173}$$

$$(\overline{\sigma}^{\mu\nu})^{\dot\alpha}{}_{\dot\beta} = \frac{i}{2}(\overline{\sigma}^\mu\sigma^\nu - \overline{\sigma}^\nu\sigma^\mu)^{\dot\alpha}{}_{\dot\beta} \ . \tag{174}$$

Complex conjugation:

$$(\psi\theta)^\dagger = \overline{\theta\psi} \ , \tag{175}$$

$$(\psi\sigma^\mu\overline{\theta})^\dagger = \theta\sigma^\mu\overline{\psi} \ , \tag{176}$$

$$(\psi\sigma^{\mu\nu}\theta)^\dagger = \overline{\theta}\overline{\sigma}^{\mu\nu}\overline{\psi} \ . \tag{177}$$

Derivatives:

$$\frac{\partial}{\partial\theta^\alpha}\theta^\beta = \delta_\alpha{}^\beta \ , \quad \frac{\partial}{\partial\theta_\alpha}\theta_\beta = \epsilon^{\alpha\gamma}\epsilon_{\beta\delta}\delta_\gamma{}^\delta = -\delta_\beta{}^\alpha \ , \tag{178}$$

$$\frac{\partial}{\partial\overline{\theta}_{\dot\alpha}}\overline{\theta}_{\dot\beta} = \delta^{\dot\alpha}{}_{\dot\beta} \ , \quad \frac{\partial}{\partial\overline{\theta}^{\dot\alpha}}\overline{\theta}^{\dot\beta} = \epsilon_{\dot\alpha\dot\gamma}\epsilon^{\dot\beta\dot\delta}\delta^{\dot\gamma}{}_{\dot\delta} = -\delta^{\dot\beta}{}_{\dot\alpha} \ . \tag{179}$$

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